CONFLICT ANALYSIS TOOLS IN SITUATIONS RELATED TO NASH'S BARGAINING PROBLEM

Radim Valenčík*, Benedikt Frank, Bastian Schulz

University of Finance and Administration, 10100 Praha 10, Estonska 500, Czech Republic

Abstract

The paper is based on the interpretation of the causes of differences between the assumed and empirically observed decision-making of players in the ultimate game. It follows the identification and definition of the phenomenon of positional investing, which allows to transform a property or income advantage into an improvement of the position of one of the players. An important step in describing the dilemmas arising in conditions where positional investing occurs is to define the function (in the simplified case, the line) of positional neutrality. To do this, it is necessary to extend the assumptions of Nash's bargaining problem. One of the most important applications is in the area of reforms aimed at more fully exploiting opportunities to develop, preserve and apply human capabilities according to their rate of return.

Keywords: *Nash bargaining problem, game theory, ultimatum game, positional investment neutrality function*

1. INTRODUCTION

The Nash bargaining problem from cooperative game theory is one of the popular and useful tools for describing situations where players share the outcome of a joint action, due to its generality and illustrative nature. However, in its many applications to the analysis of real situations, it has gradually become clear that even when all players can get along, conflicts can arise, and that analyzing the causes of conflicts and finding ways to resolve them both enriches the theory and yields a number of practical recommendations.

Building on a paper presented at the Economy & Business 2021 conference (Valenčík, Černík 2021), we show one of the original conflict analysis tools of the above type, which we called the neutrality function (in simplified form, the line) of positional investing, explain how and in what ways it extends the original Nash bargaining problem, and then outline possible applications.

We introduce the problem with an ultimatum problem, showing the importance of positional investing in interpreting the apparent differences between predicted and empirically observed player behavior. To express the role of positional investing, we will demonstrate the original tools of analysis, outlining the range of problems that can be made visible and solved through them.

2. MATERIALS AND METHODS

2.1. Materials

The aim of this paper is to present the neutrality function of positional investing as a tool for analyzing conflicts arising in problems that are extensions of Nash bargaining problems. The object of the investigation is to express and analyze the role of positional investing, i.e., such use of income earned by some player or some players in a particular game that affects the outcome of a subsequent game, where some player or some players can use this income to gain an advantage over another player or other players.

2.2. Methods

As methods we will use:

- Interpreting the apparent differences between the predicted and empirically observed behavior of players in the so-called ultimatum game.
- Analyzing the assumptions of the Nash bargaining problem and extending this problem to situations where the distribution of the gains from joint action can be a source of conflict.
- Identifying and defining the concept of "positional investing".
- Developing and presenting the neutrality function (line) of positional investing as an important tool for analyzing conflicts arising in games that are some extensions of the Nash bargaining problem.

3. THE APPARENT DIFFERENCES BETWEEN PREDICTED AND EMPIRICALLY OBSERVED PLAYER BEHAVIOR IN THE ULTIMATUM GAME

The goal of this section is to use a relatively well-known and easily imagined ultimatum game to show how the phenomenon of positional investing operates when one decides to share the payoffs from a joint action.

The ultimate game is a sequential game with two players. It is usually implemented in the context of economic experiments. The first player receives, without merit, a certain amount of money (real or fictitious, depending on the experimenter's options) and has to propose its division between himself and the second player. The second player has information about the wealth to be distributed and has the option to accept or reject the proposal. If he rejects it, then neither player receives the payoff; after acceptance, the payoff is divided according to the proposal.

Based on backward induction and the assumption of the other player's rationality, the first rational player should conclude that even with the smallest possible offer of a split, his proposal will be accepted. Thus, the first player should maximize his utility by offering the smallest indivisible monetary unit, and the second player maximizes it by accepting any positive value of the proposal.

Experiments show that the second player in line will refuse to maximize his utility if he considers the circumstances unfair. As a rule, the first player realizes this as well, so he does not maximize his utility by the seemingly rational offer of the smallest possible share, but by the offer that represents the lowest share that the second player still considers fair (which he usually estimates, since he does not have complete information here). Players most often offer a share of 40-50% and these offers are almost always accepted, while offers of less than 20% share are almost always rejected (Güth, Kocher 2014). The term "seemingly rational" is used here deliberately. Players are typically aware of the game contexts and project these into their decision-making. The decisions they make on this basis arguably cannot be described as "irrational", but rather as using context-aware rationality.

This example is the best way to understand social position investing, or positional investing, which will play an important role in our later considerations. What we subjectively experience as, for example, unfairness (in a 90:10 split, but also 70:30) is the experience-calibrated fear that a property advantage can be exploited against us in the next, downstream game. In reality, the distribution of payoffs in a given round can have a substantial effect on what happens in the future.

All of us, without realizing it, evaluate expected final states in some way. We can be wrong, we can be very imprecise, but it is better than thinking myopically. Among others, the notion of myopic and non-myopic ("myoptic" and "non-myoptic") equilibria is one of the concepts that uses a certain direction of extension of Nash's bargaining problem (Madani, Hipel 2011).

An important implication: once the distribution of the outcomes of a joint action has an impact on the future evolution of the relationships between the participants in the joint action, we need to consider the expected final states. If one of the participants can use the advantage created by the division to

improve its position, i.e., when dividing in subsequent games, this is one of the cases of positional investing that we will focus on.

The ultimatum game is one of the simplest cases of Nash's bargaining problem. We will analyze the Nash bargaining problem under the lens of the contextual perception of the ultimatum game presented above.

4. THE BASIC TYPE OF NASH BARGAINING PROBLEM AND ITS EXTENSION

The Nash bargaining problem is a very generally formulated mathematical problem that has a huge number of important interpretations. Concrete cases are, for example, every act of exchange, every creditor-debtor relationship, most contracts, but also the division of profits among partners, of loot among robbers, etc.

The Nash bargaining problem is determined by the disagreement point d (the initial situation) and the set of admissible payoff distributions S. Usually only positive values are considered and the set S is taken to be compact and convex. Then the problem can be illustrated by the following figure.

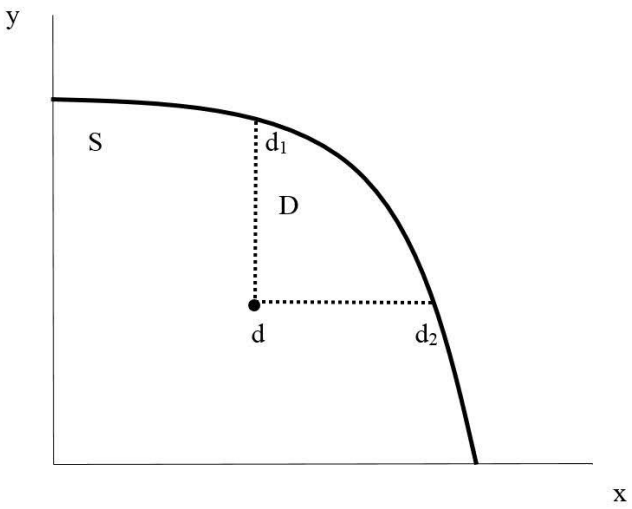

Fig. 1. Basic type of Nash bargaining problem

Source: own modification of a commonly used chart

x, y player payouts

S set of available payouts

d point of disagreement (starting point)

D the set of improvements in player payouts compared to the point of disagreement

(defined by the area between points d, d1, d2)

d1d2 (part of the boundary of the set D between points d1, d2) Paretooptimal points (points that satisfy the axioms of individual rationality, collective rationality and reachability)

The solution to Nash's bargaining problem is usually understood as looking for axioms that would allow one to uniquely determine the distribution of increased payoffs relative to point d (Nash 1950a, 1950b, 1953).

The formulation of the standard situation through the generalized Nash bargaining problem is very useful in many respects. First of all, it allows us to distinguish and define the different situations we

encounter in reality. In some cases, this distinction can be carried through to the formulation of axioms clearly defining the solution (Nash solution, Kalai-Smorodinsky solution, Raiffa sequential solution, etc.), in other cases we have at least a certain concept that we can compare with what we encounter in the field. Even this rather intuitive use of a concept corresponding to Nash's bargaining problem is often useful (Binmore, 1994), (Binmore, 1998), (Binmore, 2007), (Binmore, 2009), (Carmichael, 2005).

As in the case of the ultimatum game, in the general formulation of Nash's bargaining problem, one can ask a question related to the actual behavior of the players: will any player refuse to take joint action if he does not think the distribution of payoffs is fair?

For a long time, the prevailing belief was that if both players get better, then there is no reason not to make a deal. It is only recently that a few articles have appeared (rather scattered, both in terms of countries and the focus of the journals) that highlight this issue (Thomson, Lesenberg 1989), (Thomson W 2010).

One of the reasons for overlooking the problem outlined above has been the conception of the Nash bargaining problem as an "arbitration problem", i.e., a problem where an "independent arbitrator" presents a proposal for partition to the dividing parties.

Let us first look at the problem from the position of general methodology. Whenever a theory encounters phenomena that cannot be explained (interpreted, solutions found to the problems associated with these phenomena within the framework of the theory or the model of reality used), it seeks an overlap with the existing theory. In any particular case where the problem of overcoming an existing theory is addressed, there is a plethora of possible avenues. Only when a new theory is born does it seem in retrospect that everything was simple and clear from the beginning. How to extend Nash's bargaining problem to respect the fact that not every payoff distribution may be acceptable to every player?

In general, the following possibilities can be mentioned:

1. The distribution is usually unacceptable because the player is considering what happens after the payoffs are distributed, i.e. we will consider the fact that players are not myopic:

- Conceive of the game as a multi-round game, seeking a model of a multi-round game where the player who sees "more moves" ahead will have an advantage,
- or consider a situation where players are able to somehow, even if inaccurately or even sometimes incorrectly, estimate the "end states" and make decisions based on that estimate,
- or consider a situation where players take advantage of psychological factors like intimidate the other player (by their presence or social status)?

2. How to formulate the problem of "acceptability" of the distribution:

- Relate the problem of allocation to the consequences of allocation for individual players,
- or attempt some definition of a "fair" and therefore generally acceptable distribution,
- or determine the players' attitudes towards personal acceptance for their own benefit?

3. How to generalize Nash's bargaining problem, or what to "unpack":

- A point of disagreement in the sense that players may find themselves in a situation with payoffs that may be even lower than the point of disagreement,

- or a payout limit that may narrow or widen in certain cases,
- or by inserting some other significant element or elements into Nash's bargaining problem,
- or a combination of some of the previous options, but with emphasis on which element?

4. How to define the payoff:

- As a subjectively felt phenomenon that integrates the income aspect with its contexts (envy, etc.),
- as an objectively determined value that is fixed (from the beginning),
- or as the present value of expected future income?

5. On which area to interpret the issue:

- On the example of two equal players,
- or on the example of more players and their interrelated, interacting behaviour (e.g. in the case of unequal perception of fairness),
- or on the example of unequal players, where one can take advantage of his superiority over the other, (for example in a family like father and young reliable child),
- or on the example of the different wealth of people and the possible reliability between them,
- or educational/knowledge advantage between two players,
- or experiental advantage, where one of the players has more experience than the other,
- or on the example of war or the threat of war,
- or the relationship between the owner of labour and capital, or between the employee and the company,
- or to look for a universal interpretation that is not linked to the solution of this or that real problem,
- or to seek some compromise between dealing with more specific cases that have practical relevance and the ambition to deal with the problem at the most general level?

Identifying the neutrality function of positional investing, as we will show in the next section, is the key to choosing the most appropriate combination among the alternatives on offer.

5. THE NEUTRALITY FUNCTION OF POSITIONAL INVESTING

Let us again ask the most important question: Why are some payout distributions unacceptable to some players? The answer to this question puts a tool in our hands that takes our understanding of social reality a great deal further.

We know that the results of payout distributions in a particular round may not be neutral to what happens in the future. For example, one player may gain an advantage over another player. In general, we can call this phenomenon positional investing, i.e., the possibility of turning a higher payoff into an advantage that discriminates against the other player. The mechanism of this conversion is to increase the payoff of one's own investment opportunities by reducing the payoff of the other player(s)' investment opportunities, while reducing the efficiency of the relevant economic system. This defines positional investing as such (Valenčík, Černík 2021, p. 226).

The distribution of payoffs that allows such a situation to arise will be unacceptable to the player who may be affected by this distribution and who is not myopic. But how to express this fact through theoretical tools (Dhami 2017), (Loomes 1999)?

An analogy is offered with one of the most important tools used in microeconomics: consumer preferences are expressed through indifferences, i.e. indifference curves. An indifference curve is a set of points at which a consumer derives the same utility from the consumption of two goods (consumed in different combinations). In the logic of the case, this curve shows the amount of one good that the consumer is willing to exchange for the other good in order to retain the same utility.

In our case, we can assume that there are points in the payoff distribution at which neither player gains an advantage over the other in the sense that he can turn his payoff into an advantage that discriminates against the other player relative to the initial state.

It can be assumed that each payoff point of one player corresponds to a corresponding payoff point of the other player, where neither player can use the advantage hidden in the payoff to gain a positional advantage. If each such point of one player corresponds to a corresponding point of the other player, we can speak of a position neutrality function.

It can be argued:

1. This point may not exist, or on the contrary, there may be more than one of these points (and then we usually cannot speak of a "function" in the mathematical sense).

2. Players may see the situation differently, then each would be working with a different idea of the positional investing neutrality function.

Both objections are relevant, but we will see that they can be very well handled and that they are in some sense mutually eliminating, or the positional investing neutrality function is the key to dealing with these objections.

Let's get a more concrete idea of what the set of neutral points that represent a particular equilibrium in positional investing looks like. It can be viewed, for example, as a certain function that assigns each player's payoff to the other player's payoff, for example, in an implicit form as a function:

$N(x, y) = 0$

or in explicit form from the perspective of one of the players as:

$y = n(x)$

i.e., what payoff y must player Y have in order not to be disadvantaged but not to be advantaged by the payoff distribution in this round compared to player X in subsequent rounds.

This function, as mentioned above, is called the payoff distribution neutrality function.

For purely terminological reasons, we will call the payoff distribution neutrality function the positional investing neutrality function because, as we will show, it is the phenomenon of positional investing that allows a player to use his payoffs received in the form of income to influence the payoffs in the next round.

Recall once again that the key to describing the phenomenon of positional investing formulated in this way is very similar to the logic underlying the indifference curve. In the case of the indifference curve, it is a matter of expressing preferences through indifferences; in the case of positional investing, we describe its effect through the "curve" (function) of its neutrality (i.e. by identifying cases where its effect is zero, in a situation where its effect may be non-zero if the distribution of payoffs were outside the neutrality function of positional investing).

Very trivially, the characterization of positional investing can also be formulated as follows: If one is to share something with someone else, one must first make a judgment as to whether what accrues to one is sufficient so that what the other gets cannot be used against him.

The neutrality function of positional investing can be thought of as follows:

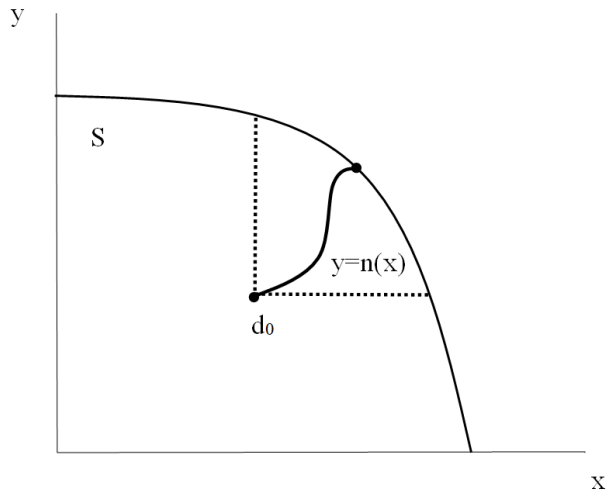

Fig. 2. The neutrality function (or set of neutral points) of positional investing

Source: Own creation

If there is a position neutrality function, the optimum point in solving the Nash bargaining problem is uniquely determined as the maximum of its attainable value in the set S.

Incorporating the positional investing neutrality function into an apparatus that allows one to analyze the distribution of income among the participants in a joint action is much more important than it may seem at the moment. It will answer, among others, the following questions:

- What was the main mistake made by the followers of Nash and why did they make it? Or vice versa, which advantages did they use?
- What is the source of conflict in the distribution of the results of joint action?

Before going into answering these questions, let us make a technical simplification.

To make the concept of graphical representation of positional investing easier to work with (to get a better idea of what is going on), we will treat the neutrality function of positional investing as linear (which is an insignificant simplification in terms of conclusions, but very useful in terms of explaining the issue):

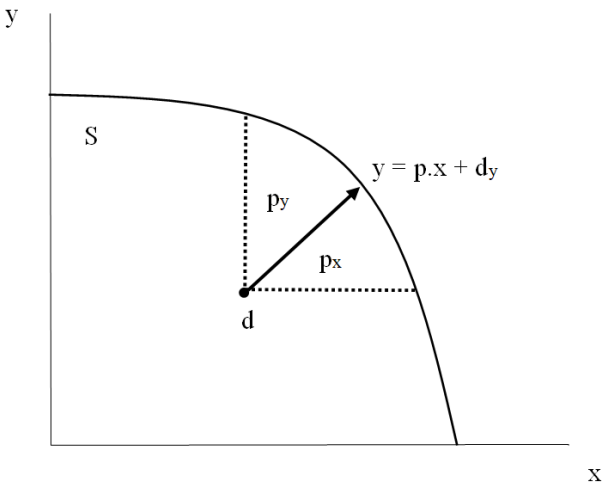

Fig. 3. The linear neutrality function (or set of neutral points) of positional investing (i.e. the neutrality line of positional investing)

Source: Own creation

$y = p.x + d_y$ linear function of positional investing neutrality

p_x points to the right of this function are unacceptable to player Y

p_y points to the left of this function are unacceptable for player X

Introducing this simplification allows us to talk about the positional investing neutrality line and, in a much more illustrative position, allows us to raise the question of what determines the slope of the positional investing line and what the consequences of changing the slope of positional investing are.

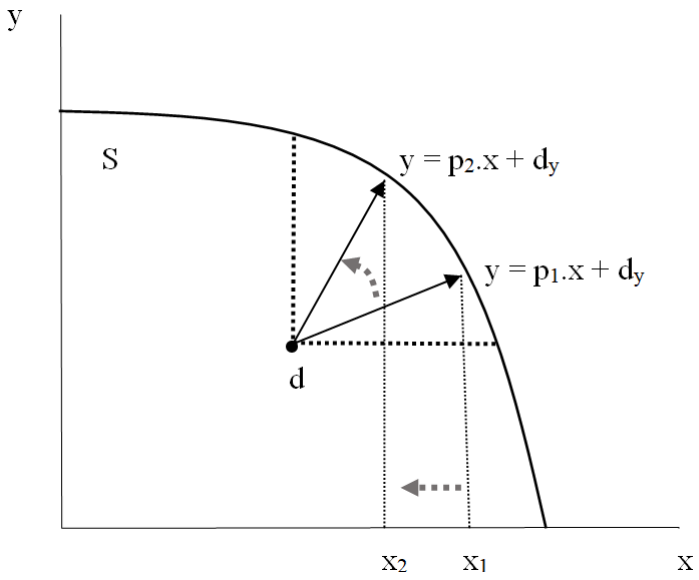

Fig. 4. Interpreting a change in the slope of the positional investment line

Source: Own creation

Assume that only player Y can take advantage of positional investing.

In our case in the figure: $x_2 < x_1$

This means that the player who does not have the option of positional investing only needs to gain less after the change in the slope of the positional investing line presented above to avoid a deterioration in positional investing. This does not mean that he gains less, but that the danger of positional investing has decreased.

One interpretation, as we will see subsequent, is the way to answer the question of how to reduce the social tension that results from the unequal distribution of wealth in society, more precisely from the possibility of turning a property advantage into an advantage for some over others. It is a question that is very important at the present time.

Nash's bargaining problem assumes that everyone (the players, and possibly the arbitrator) sees the situation in the same way. This is, of course, an overly restrictive assumption, but on the other hand, without appropriate interpretations of the more general approach, we would make no further progress. Here, however, one interpretation is directly imposed.

In addition to the initial situation already mentioned, where both subjects see the parameters of the game in the same way, we can consider two different cases:

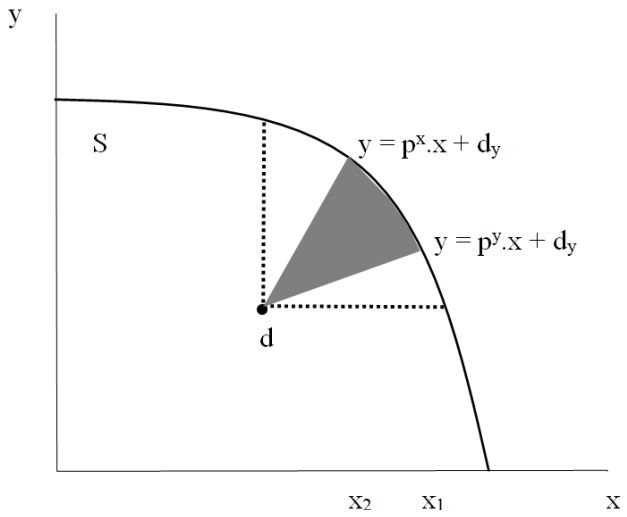

Fig. 5. The case where there is room for negotiation

Source: Own creation

$y = p_x.x + d_y$ is the line of neutrality of positional investing as seen by player X

$y = p_y.x + d_y$ is the line of neutrality of positional investing as seen by player Y

The lines differ in slope, which we refer to by superscripting the coefficient p.

The grey shaded area is the area in which all payoff distributions are acceptable to both player X and player Y.)

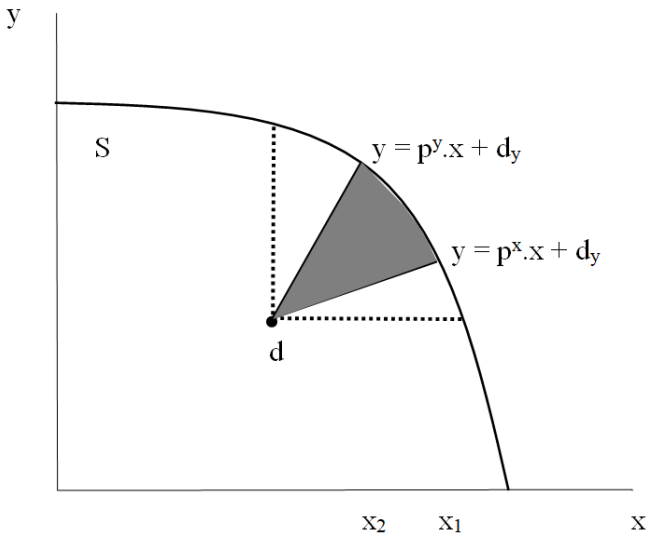

Fig. 6. The case where there is no room for negotiation

Source: Own creation

Here the same variables figure, but the slopes of the position investing neutrality lines are flipped. In the grey shaded area, and even more so outside it, there is no payoff distribution acceptable to either player X or player Y.)

In this case, the players not only fail to agree, but necessarily clash. To describe this conflict, we need to add to the arsenal of theoretical tools we use to describe this situation. These tools are based on a multi-point extension of Nash's bargaining problem.

As we have already stated, the solution to the original Nash bargaining problem assumes that both players see the situation in the same way. This is a strong enough assumption that is usually not met in practice. The theory has dealt with it in the domain in an instructive way: it has assumed the role of an independent arbiter who will ensure that both parties see the same thing, and that this (his vision) will be recognized by both parties, just as his decision will be. Procedures based on the use of the neutrality function of positional investing offer a way to apply the tools of cooperative game theory much more widely.

6. DISCUSSION AND RESULTS

In discussing the benefits and how to apply practices and tools based on the position neutrality function of investing, we will focus on reforms that create greater equality of opportunity, i.e. that allow investment opportunities to be more fully exploited according to their rate of return, including those related to the acquisition, retention and application of human skills. Such reforms will both increase the efficiency of the system (thereby allowing more wealth to be distributed) and reduce the need for positional investment (which we illustrate in the figure below by changing the slope of the positional investment line). If the necessary reforms do not take place, the role of positional investment is strengthened and the economic system degenerates or misses an opportunity for evolutionary development (Palacios 2007).

Through the apparatus we have created, which can be further developed, we can distinguish between two opposing alternatives of development: either society evolves towards a fuller exploitation of investment opportunities linked to the development of human capabilities (human capital acquisition), or it evolves towards a more limited exploitation of investment opportunities linked to the development of human capabilities (human capital acquisition). This can be illustrated by the following pair of figures.

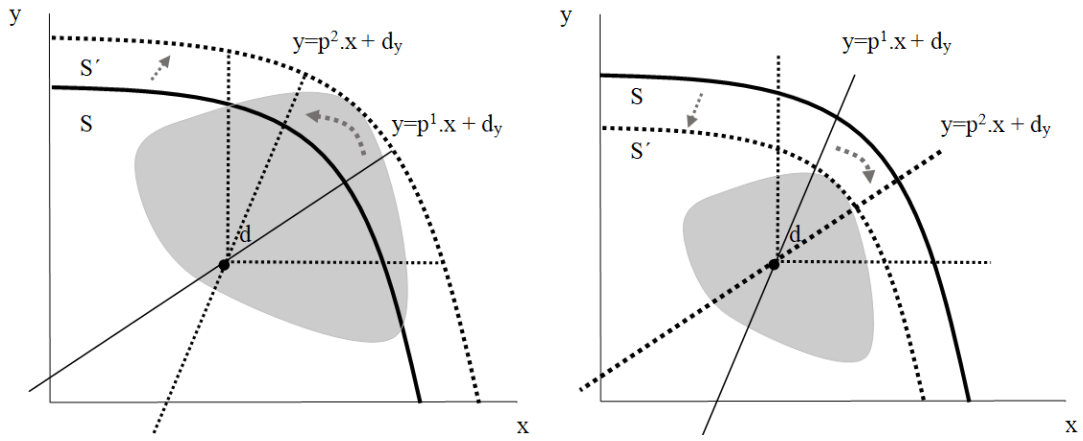

Fig. 7. Opposing alternatives for the development of society

Source: Own creation

In the first case (on the left), the economic system will be more efficient, more wealth can be distributed (in the figure expressed as a shift of the boundary of the possibilities of wealth distribution from the bold line to the dashed line) and at the same time the effect of positional investment on the

possibility of social rise independent of the initial wealth ratios will be reduced (in the figure this is expressed by rotating the line $y = p_1.x + d_y$ to the position $y = p_2.x + d_y$, i.e. counterclockwise).

In the second case (on the right), there will be a loss of efficiency, less wealth can be distributed (in the figure expressed as a shift of the frontier of the possibilities of wealth distribution from the bold line to the dashed line) and at the same time the effect of positional investment on the possibility of social rise independent of the initial wealth ratios will increase (in the figure this is expressed by rotating the line $y = p_1.x + d_y$ to the position $y = p_2.x + d_y$, i.e. clockwise).

In line with the above pair of figures, further questions arise:

- Why are reforms that would make a positive alternative development possible not only not being implemented but also not being discussed in the public space? It seems that especially nowadays with several crises, that the public has to deal with, the public should be more invested and look for sustainable solutions. The public space should focus on reforms that would improve the life of their population.
- Why is development in the individual countries moving more towards degeneration, i.e. deepening economic and, consequently, social segregation? Especially in times like right now, it seems logical that countries would work together and support each other instead of not being open for cooperations or closing them down.

Answering these questions is not easy, because, according to what we can see in the figures above, if real reforms were to be implemented, virtually everyone would be better off, including those who benefit from positional investment. Uncovering the non-cooperative games that derive from the conflicting alternatives is the current task of further investigation.

We open the way to this investigation by interpreting the causes of the differences between the predicted and empirically observed decision making of players in the ultimatum game. In doing so, we have taken advantage of identifying and defining the phenomenon of positional investing, which allows one to convert a property or income advantage into an improvement in the position of one of the players. The most important step in describing the dilemmas arising in conditions where positional investing occurs was to define the function (in the simplified case, the line) of positional neutrality. To do this, it was necessary to extend the assumptions of Nash's bargaining problem. One of the most important applications has been in the area of reforms aimed at more fully exploiting opportunities to develop, preserve and apply human capabilities according to their rate of return.

7. CONCLUSIONS

The problems of social and economic development today can be viewed in different ways. There are different perspectives on how the problems arose and how to address them. The consensus is that these are serious problems and that we do not yet have a sufficiently clear and commonly shared understanding of how to address them. Nevertheless, it seems logical that we need to find common solutions, especially in times like right now, where it appeared like the world came together to fight the corona crisis or the financial struggles regarding the Russia-Ukraine-war. That might be a milestone when it comes to a cooperation between all countries over the world that is a good first step towards a better future while working together.

The paper is framed within an approach that emphasises the growing role of services that are productive in the sense that they contribute to the acquisition, retention and application of human skills. On this basis, the innovation potential of society is increased, which is the fundament for dynamic economic growth and, at the same time, an environmentally friendly relationship.

In order to raise the role of productive services to the level of the dominant economic sector, a complex of reforms is necessary to substantially increase the rate of exploitation of investment opportunities linked to the acquisition, retention and application of human skills, according to the degree of their profitability. This is an area in which the role of the social investment and social insurance system (financing education, health care and old-age pensions) is combined with that of the

financial markets. Not only the implementation but also the design of such reforms faces various barriers, the most important of which are related to the role of positional investment.

Defining the neutrality function of positional investing is an important tool for uncovering and analysing the dilemmas involved in the design and implementation of complex reforms in social investment and social insurance systems.

The future will tell if it is possible to overcome the currently enormous problems in social and economic development. Yet, it should be mentioned that countries need other countries to learn, improve and share problems and success stories. The same applies to individual people. The long-term perspective will always be to work together instead of working alone and achieving short-term goals that are not sustainable. It will be interesting to see in the future if the above-mentioned problems will be solved and how. Time will tell.

ACKNOWLEDGMENTS

This paper has been prepared within the project *"Wealth and poverty as a problem in terms of economics of productive consumption"* University of Finance and Administration (2022)

REFERENCES

1. Binmore, K 1994, "Playing Fair: Game Theory and the Social Contract I". Cambridge (Ma): MIT Press.
2. Binmore, K 1998, "Just Playing: Game Theory and the Social Contract II". Cambridge (Ma): MIT Press.
3. Binmore, K 2007, "Playing for real: a text on game theory". Oxford: Oxford University Press.
4. Binmore, K 2009, „Game theory and institutions". ELSE Working Papers, No.331. London: ESRC Centre for Economic Learning and Social Evolution.
5. Carmichael, F 2005, "A guide to game theory". Harlow: Pearson Education Limited
6. Dhami, S 2017, "The Foundations of Behavioral Economic Analysis", Oxford: Oxford University Press.
7. Güth, W, Kocher, MG, 2014, "More than thirty years of ultimatum bargaining experiments: Motives, variations, and a survey of the recent literature", *Journal of Economic Behavior & Organization, 108,* vol., pp. 396–409.
8. Loomes, G, 1999, "Some lessons from past experiments and some challenges for the future", *Economic Journal*, vol. 109, no. 2, pp. 35–45.
9. Journal, 109(2):35-45. 1999.
10. Madani, K, Hipel, W. 2011, "Non-Cooperative Stability Definitions for Strategic Analysis of Generic Water Resources Conflicts", *Water Resources Management*, Dordrecht, vol. 25, no. 8.
11. Nash, J 1950a, "The Bargaining Problem", *Econometrica*, vol.18, pp. 155-162.
12. Nash, J 1950b, "Non-Cooperative Games". Ph.D. Dissertation, Princeton University.
13. Nash, J 1953, "Two person cooperative games". *Econometrica*. Vol. 21, pp. 128–140.
14. Palacios, M. Investing in Human Capital: A Capital Markets Approach To Student Funding. Cambridge: Cambridge University Press. 2007.
15. Thomson W, Lesenberg T 1989, "Axiomatic theory of bargaining with a variable number of agents", Cambridge University Press, Cambridge.

16. Thomson W 2010, “Bargaining and the Theory of Cooperative Games: John Nash and Beyond”, Edward Elgar, Cheltenham, UK.
17. Valenčík, R, Černík, O, 2021, “The equality in society and a multipoint extension of Nash bargaining problem”. *Economy & Business*. Burgas, Bulgaria: International Scientific Publications, vol. 15, pp. 221-232.