# *Nash Equilibrium in Redistribution Systems (Calculation, Weight, Application)*

# *Nashova rovnováha v redistribučních systémech (výpočet, význam, využití)*

**PETR BUDINSKÝ, RADIM VALENČÍK**

The more the company is able to appreciate performance of its employees the higher its output. This applies in general to other social systems as well - public administrative institutions, organizations, political competitors, public associations, regional self-governing bodies on different levels, the private and the public sector etc. The above principle also works in the opposite direction: The bigger the collision between cash distribution inside the social system and appreciation of performance of those creating the system, the lower the overall performance of the system as a whole. A typical case of redistribution inside such systems is formation of a certain coalition using its dominant influence for redistribution of the means acquired by the company for the benefit of the members of the coalition. This also applies to administered organizations where the person deciding about distribution of gains is appointed for the position and enjoys unlimited or at least significant powers. Here also various informal coalitions are formed with the abovementioned goal.

The theory of redistribution systems may be applied to resolution of this issue as one of the cases of the general theory of game. The theory represents an original approach developed by a team of the College of Finance and Administration and applicable in many different areas. For the purpose of analysis of standard situations occurring in the redistribution systems there is a formalized model of elementary redistribution system with three players (A, B, C) whose performance is distributed in the ratio of 6:4:2. Each of the players possesses the same power to affect the result (influence equal to 1).[1] One of the first steps of the redistribution system analyses was definition of the redistribution equation describing all options of payment distribution in the elementary redistribution system. For the purpose of the elementary redistribution system the equation can be defined as follows:

$$\mathbf{x + y + z = 12 - \eta.R(x - 6, y - 4, z - 2)}$$

where:

**x + y + z** is the sum of real pay-offs of individual players,

12 is maximum pay-off which could be divided, if output of the redistribution system was maximal, which means that no distribution would be taking place and dividing of pay-offs would be according to performance, η is coefficient of lowering performance,

1 *For details see Wawrosz (2007).*

**R(x - 6, y - 4, z - 2)**

is function of length of division of real pay-offs according to performance.

The redistribution equation may be interpreted as follows: the amount to be distributed among the players equals the maximum amount that the players might distribute among themselves reduced by the value of their deviation from performance-based distribution. The function of distance **R** can be defined in various ways. The best way seems to be the definition based on the usual metrics defining the distance as square root of the sum of squares of the difference between the optimum payment and the actual payment (only positive values being taken into account):

$$\sqrt{(x - 6)^2 + (y - 4)^2 (z - 4)^2}$$

A professional mathematical analysis of the elementary redistribution system is extremely important for two reasons. One is its relevance for analyses of various types of the elementary redistribution system applications and the other its relevance for analyses of how the simple elementary systems are chained into more complex ones. Every equilibrium within a simple redistribution system is unstable and that leads to chaining of simple systems into hierarchical and network structures.

This analysis shows that what might appear to be an external influence (such as personal sympathies etc.) often is determined by the very parameters of the system.[2] Further to this issue there is a very interesting recently developed apparatus based on a computer model, used for simulation of a number of situations opening the way to analyses of the more hidden layers of the issue. The model allows for descriptions of various types of negotiations and their results, which display as negotiation trajectories in the redistribution area.

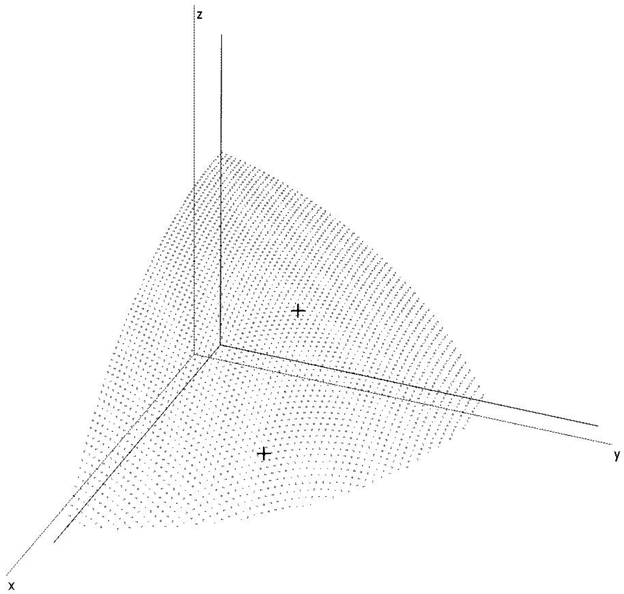
Diagram 1 shows an example of computer plotted redistribution area for the value of the coefficient of performance decrease η equal to 0.5 and R defined as square root of the squares of the difference between redistribution and performance-based remuneration. The bottom cross sign marks a point with coordinates (6; 4; 2), i.e. the point of payment distribution based on performance, and the top cross sign marks the point of equal payments to all players, which in the given case is about 3.51, i.e. a point with coordinates (3.51; 3.51; 3.51). Every redistribution plane must intersect both points. In the point with coordinates (6; 4; 2) the sum of payments to all players is the highest. The farther from this point the lower the sum of the payments.[3]

---

2 *M. Maňas, in the context of solution of a similar task (collusive oligopoly consisting of five players) after presenting all equilibrium situations states: "Contract negotiation is usually long and if in general fatigue resulting form the protracted negotiations a contract is signed in the end, it is mostly under the effect of personal sympathies rather than on the basis of a logical consideration. " (Maňas 2002, p. 61) In the area of redistribution systems, however, we can go even further and look at the hidden causes of what appears to be personal sympathy etc.*

3 *The model works with the step equal to 0, 1. Compiled by computer science student Mr. Vávra (from IFA)*

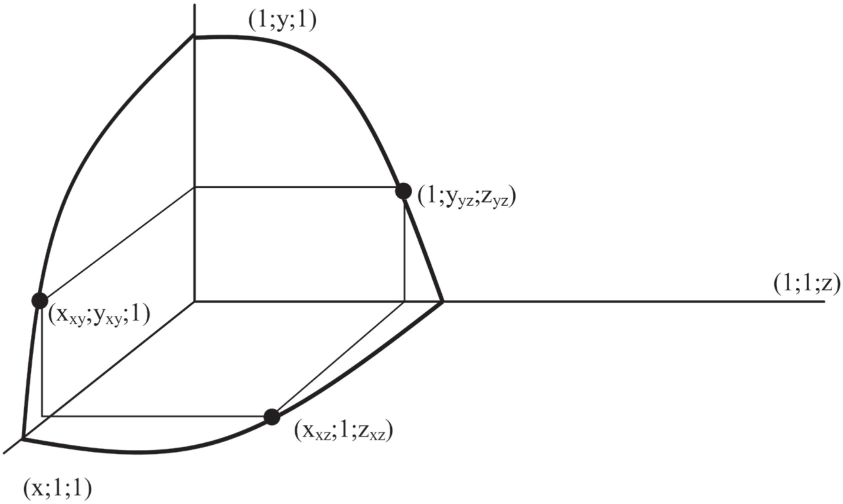
**Diagram 1** Computer-based simulation of redistribution surface




Let us look at one particular and very practical conclusion that can be drawn on the basis of the analysis of what happens in the redistribution area. With regard to coalition forming the redistribution area looks very symmetrical at first sight. The player with the top performance (A) can form a coalition with the average player (B) and both of them can improve their earnings at the cost of the weakest player (C). Similarly player B can form a coalition with player C and improve its earnings at the expense of player A. And then there is the third option of a coalition between players A and C against player B. In the case of a coalition between players B and C and improvement of their earnings at the cost of the top performance player will be the highest (for they can get most of the player with the top performance). At the same time this will result in the largest decrease of the overall performance of the system. The conclusion drawn from the above is very important. Under very general assumptions actual systems will tend to "drop" to a situation when the mediocre ally with the weakest and control the system (institution, organization etc.) including distribution of earnings within the system.

The strongest player, however, is not completely helpless and can do something against this development of the situation. He can offer more to the weakest player than what the weakest player would achieve by alliance with the mediocre. Or instead of the promises to the weakest player the strongest player may ally with the mediocre player and they can improve their situations in comparison to the situation under agreement with the weakest player. In the logic of the case the agreement of a player with the player other than the member of the original coalition and the resulting redistribution of payments represents sacrificed opportunity. In he case of the offers described above a equilibrium can be calculated.

**Diagram 2:** Equilibrium in negotiations by using pandering method:




The key to finding a equilibrium in the case of negotiation with offer is represented by the following consideration:

- If an agreement has been made between the weakest and the mediocre player and the parameters of the agreement are $(1;y_{yz};z_{yz})$, then with regard to payment to the weakest player there is a equilibrium between this agreement and the agreement between the strongest and the weakest payer with the parameters $(x_{xz};1;z_{xz})$.
- In this case the following must be true: $z_{yz} = z_{xz} = \text{def: } z_p$ (the value must be the same whether resulting from negotiation between the weakest and the mediocre player or from negotiation between the weakest and the strongest player and therefore may be $z_p$ in both cases, with the p index derived from the word promise (offer).
- The same obviously applies in the case of the other agreements, i.e.:

$x_{xy} = x_{xz} = \text{def: } x_p$
$y_{xy} = y_{yz} = \text{def: } y_p$

This relation implies following equation system:

$$1 + y + z = 12 - \eta.R(5;\ y - 4;\ z - 2)$$
$$x + 1 + z = 12 - \eta.R(x - 6;\ 3;\ z - 2)$$
$$x + y + 1 = 12 - \eta.R(x - 6;\ y - 4;\ 1)$$

There are three independent equations with three variables whose solutions represent the sought for values. What is the purpose of this solution? The solution points out three equilibrium points with coordinates:

$(1; y_p; z_p)$ – with player A outside the coalition and discriminated
$(x_p; 1; z_p)$ – with player B outside the coalition and discriminated
$(x_p; y_p; 1)$ – with player C outside the coalition and discriminated

Let us call equilibriums of this type discrimination equilibriums. They can already be used for calculation of the Nash equilibrium. The ratio of mean payments to the players (the sum of value 1 and the two values corresponding to the victorious coalition, all divided by three) is to be entered in the redistribution equation. The solution to this equation is the Nash equilibrium.

The definition of Nash equilibrium is not simple and some monographs of textbook type even include certain inaccuracies. A detailed account is presented by Carmichael (2005)4. Her detailed definition reads: "In Nash equilibrium the players in the game select strategies to each other that are the best for themselves. But not every Nash strategy applied by the individual player is the best answer to every other strategy applied by the other players. Nevertheless, if al players in the game play Nash strategies, none of them is inclined to do anything else."

If any player wants to improve its pay – whether by requirement for a higher pay or by a promise to and a coalition with another player with discrimination of the third player

- he will achieve the very opposite, his situation will get worse.[5]

It is clear that if players B and C manage to exclude player A from the possibility to negotiate, their reward will be higher than in the case of the above calculated Nash equilibrium. This leads to a conclusion important for the practice, which is that in real systems you can come across cases when the top performance player is in advance deprived form the possibility to negotiate who will assert himself in the given system.

The principles of calculation of Nash equilibrium can be transferred to situations representing an extension of ERS, for example if:

- There is some pressure from competitors exerted on the system.
- The system develops (grows) in time and compares to other systems.
- There is a back effect of the income on the negotiation positions of the players (their ability to affect the redistribution).
- There is inter-organisation migration.
- Etc.

In all these cases it is possible to express the effect of the extended assumptions on the negotiation positions of the players and in which directions the discrimination equilibriums and the Nash equilibrium will be shifted. Discrimination equilibriums of a similar type and Nash equilibrium even exist in more complex systems with relationships between areas with effective competition pressures, and areas with limited competition pressures and

4 *Carmichael. (2005, p. 36).*

5 *For details see Valenčík et al. (2007).*

with the option of inter-organisation migration. Generally speaking the proof of existence and demonstration of the possibility of calculation of Nash equilibrium in an elementary redistribution system in the key to identification, description and possibly calculation (if you quantify the system parameters) of Nash equilibrium for more complex redistribution systems. It is possible to provide a general methodology of analysis of effects of various factors on the shifts of the individual types of the equilibrium.

Identification, description and possible calculation of Nash equilibrium in redistribution systems are crucial. As the discrimination equilibriums and the Nash equilibrium are statuses with very close probability, but significantly different positions of the individual players, important rules can be derived for combinations or mergers of simpler redistribution systems to more complex wholes and rules of development of social networks acting between various redistribution systems. Research into this is quite intense. The results of the research are continuously published on www.vsfs.cz/vyzkum-a-projekty/seminar/.

The theory of redistribution systems offers broad possibilities for international cooperation, among other things in areas such as doctoral study. The solutions of the individual issues may focus on achievement of original results with regard to development and application of a mathematical apparatus specific for the given area (including one not yet applicable in economic disciplines) as well as with regard to a number of important and socially relevant practical applications.


## Abstract
A redistribution system is any social system as organization, company or institution is where the redistribution of payments of players in comparison with their performance happens. The very important role is played by constitutions of alliances created in this system. Modeling of essential three players redistribution system as well as identifying and calculating the forms of equilibrium (discriminatory or Nash) provide the key for analysis of a real individual or group behavior including problems of its stability and chaining of other social systems. The theory of redistribution systems, as an original extension, comes out of the "game theory" and it has numerous practical applications.



## Keywords
Game Theory, Theory of Redistribution Systems, Redistribution Equation, Coalition, Bargaining, Pareto Optionality, Nash Equilibrium

## JEL classification / JEL klasifikace
D01, D33, D74.



## Kontaktní adresa / Contact address
**h. doc. RNDr. Petr Budinský, CSc.**
Vysoká škola finanční a správní, o.p.s., prorektor pro obchod a vnější vztahy
(e-mail: petr.budinsky@vsfs.cz)

**doc. Radim Valenčík, CSc.**
Vysoká škola finanční a správní, o.p.s., Centrum pro ekonomické studie a analýzy
(e-mail: radim.valencik@vsfs.cz)

## Souhrn

Redistribuční systém je jakýkoli sociální systém typu organizace, firmy či instituce, ve kterém dochází k přerozdělení výplat hráčů oproti jejich výkonnosti. Podstatnou roli přitom hrají koalice, které se v takovém systému vytvářejí. Modelování elementárního redistribučního systému, který se sestává ze tří hráčů, identifikování a výpočet rovnováhy diskriminačního typu a Nashovy rovnováhy poskytuje klíč k analýze reálného chování jednotlivců i jejich skupin včetně problematiky stability a spojování různých sociálních systémů. Teorie redistribučních systémů jako původní rozšíření a aplikace teorie her má četné praktické aplikace.

## Literature

**BEDRETDINOV, R.; VALENČÍK, R.; WAWROSZ, P.** 2006. Obecná teorie redistribučních systémů. In *4. výroční konference České společnosti ekonomické*. Praha : ČSE, 2006.

**CARMICHAEL, F.** 2005. *A Guide to Game Theory.* Harlow : Pearson Education Limited, 2005. ISBN 0 273 64896 5.

**HORNIAČEK, M.** 2006. Negotiation, preferences over agreements, and the core. In *4. výroční konference České společnosti ekonomické*. Praha : ČSE, 2006.

**MAŇAS, M.** 1991. *Teorie her a její aplikace.* Praha : SNTL, 1991.

**MAŇAS, M.** 1995. Hry v ekonomické teorii. *Politická ekonomie.* 1995, roč. 43, č. 1, s. 105-112. . ISSN 0032-3233.

**MAŇAS, M.** 2002. Konkurenční a koluzívní chování v oligopolu. *Politická ekonomie.* 2002, roč. 50, č. 1, s. 37-47. 1/2002.ISSN 0032-3233.

**OSBORNE, J.** 2002. *An Introduction to Game Theory.* New York : Oxford. University Press, 2004. ISBN 0-19-512895-8.

**SEKERKA, B.** 2002. *Mikroekonomie. Matematické a kvantitativní základy.* Praha : Profess Consulting, 2002. ISBN 80-7259-030-8.

**TULLOCK, G.** 1997. *Economics of Income Redistribution*. 2. vydání. Boston-Derdrecht-London : Kluwer Academic Publisher, 1997. ISBN 0-7923-0991-5.

**WAWROSZ, P.** 2007. Investování do sociálního kapitálu, efektivnost a redistribuční systémy. *Acta VŠFS*. 2007. roč. 1, č. 1, s. 74-102. ISBN 978-80-86754-86-4.

**VALENČÍK, R.** 2007. Diskriminační a Nashova rovnováha v elementárním redistribučním systému. In: *Efektivnost investování do lidského kapitálu. Ed. Valenčík, R. Praha: EUPRESS, 2007. ISBN 80-86754.*