Information Frictions and Monetary Policy
Informační frikce a měnová politika

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Abstract
Real effects of monetary policy depend crucially on the nature of nominal rigidities. These rigidities are typically modelled as sticky prices with explicit assumptions on either frequency of price adjustments (Calvo-style models) or on the cost of adjustment (menu cost models). However, recent empirical work cast doubts on these workhorses of standard New Keynesian models. This paper discusses another approach to nominal frictions, which is based on the assumption that agents face difficulties processing information. If, for instance, price-setters learn about an interest rate cut with a delay, then their price also responds sluggishly. This rigidity implies positive temporary effects on output and unemployment. We conclude that models based on information frictions can account for several empirical facts other model have difficulties reconciling with, such as sluggish responses of both real and nominal variables, frequent but staggered price changes or a steeper Phillips curve and higher profit losses with more volatile environments. Moreover, rational inattention provides important implications for policy.

Keywords
nominal rigidity, information frictions, monetary economics

JEL Codes
D21, D83, E31, E52

Introduction

Models that are used to assess the optimal monetary policy are typically built around certain assumptions about the nature of nominal rigidities. Different assumptions generate different results and thus also potentially drastically different prescriptions for optimal policies. This paper argues that there are several good reasons to built such models around information frictions in the form of agents' inattention. We also discuss the basic policy implications of such models.

The most common nominal friction are driven by explicit assumptions of price stickiness using Calvo-style adjustments or some form of menu cost. Bils and Klenow (2002), however, cast doubts on these assumptions by finding that individual prices do not stay fixed for long periods of time. When the models are calibrated to fit the observed frequency of price changes, then the implied real effects of nominal shocks are very small. Bils and Klenow (2002) thus motivated macroeconomists to focus on prices at the micro level, too.

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An alternative line of the modeling of nominal rigidities is based on the assumption that agents cannot attend to all the available information about new shocks. This idea was proposed by Christopher Sims, formulated in a framework called "rational inattention"(Sims, 1998, 2003). Simply put: if price-setters do not pay attention to new shocks, they can not respond to them and their prices thus stay rigid.

Mackowiak and Wiederholt (2007) showed that rational inattention can generate real effects of monetary policy. While nominal aggregates, e.g. the price level, respond to monetary shocks with a delay, individual prices change all the time, which is, however, counterfactual. This problem was resolved in Matějka and Sims (2010) and Matějka (2010a). These papers expand the approach of Mackowiak and Wiederholt by not only modeling of how much information price-setters process, but also what they process information about.

Rationally inattentive agents in these models actively seek those pieces of information that carry the most value. It turns that the implied price dynamics of such price-setters corresponds very well with the data: prices stay rigid for a while, but not for too long, and aggregates respond with a delay to aggregate shocks, which generates real effects of monetary policy.

Rational inattention describes the humans' limited ability to process information. Most of policy-related information is accessible with very little cost. We could, in principle, find out what the current federal funds rate is at every single moment, we could find out the last reported unemployment rate or the GDP growth. These pieces of informational are easily attainable at most major business magazines or on the internet. Few of us, however, do so very frequently.

Intuitively, the less the information is important for one's business the less likely it is that the individual bothers to devote time to acquire it. For example, most bankers know the current level of the funds rate at least approximately, lower number of academics know about it, and some local businessmen would not even know whether the rate moved in the last two years at all.

Although the papers above show that rational inattention can account for realistic dynamics of prices, it has been criticized on the grounds that the required amount of information price-setters process is too low. This criticism was addressed in Matějka (2010b), where the author shows that inattentiveness on the consumers' side suffices to generate rigidity of prices. The intuition for this result goes as follows: consumers who dislike processing information prefer stable prices. If prices are not stable, then the consumers have to pay extra cost finding out what the current prices are. Price-setters consider effects of their dynamic pricing strategies and choose to keep their prices relatively stable, in order to attract more consumers and induce higher sales.

This paper expands on Matějka (2010a) and Matějka (2010b). It does so by studying dynamic effects of aggregate shocks and by exploring implications of inattention in various environments. We find that, for instance, that nominal rigidity is weaker in more competitive industries or under more volatile aggregate conditions. This is particularly important at times of crises, such as in 2008 and 2009, when the resulting responses to monetary shocks can be quite different from those during normal times. When we account for endogenous choice of attention level, prices are more flexible during times of high volatility, since agents choose to process more information. This implies that monetary policy has little real effects.

These findings have numerous implications for policy, which we briefly discuss in the final section.

1 Nominal Rigidity

Rational inattention allows for endogenizing what pieces of information to process. Decisions of rationally inattentive agents are not a priori biased by a specific form of any mechanism by which they process information. All pieces of information are freely available and agents can select which of them to process. It sounds intuitive that agents want to be more aware of variables that are important to them, such as income to a household or input cost to a producer. Or for instance, a price-setter in a highly competitive industry could devote more attention to prices of his competitors than if the competition were low.

Agents not only differentiate between variables (e.g. attention to inflation or GDP, etc.), they can also pay different amounts of attention to different levels of the same variable and also different amounts of attention to the same variable under different conditions. If an agent uses a credit card, he may not pay full attention to the level of his debt and may consume some typical amount. However, once his debt approaches the credit limit, he needs to be more aware of exactly how much more he can charge on the card. Furthermore, economic actors do not need to pay much attention to slowly moving variables such as to most of the aggregate indices. The more predictable variables are, the less information needs to be processed about them. In case volatility of these variables increases, agents might check on their current values more often, to keep being informed reasonably precisely.

This section focuses on nominal rigidity. It first presents two examples of static problems studying effects of competition on what sources trigger swifter responses, on the informational content of prices and on the rate of responses. Next, we present a dynamic model studying trade-offs between attention devoted to stable aggregate variable, such as a price level, and volatile input cost, such as commodity price. We also discuss implications of varied levels of the respective volatilities, which provides us with intuition about monetary policy effects on different industries. The section is concluded with a dynamic model of an inattentive consumer.

1.1 Allocation across Variables: When Sellers Respond to Cost Shocks and When to Changes of Competitor's Prices

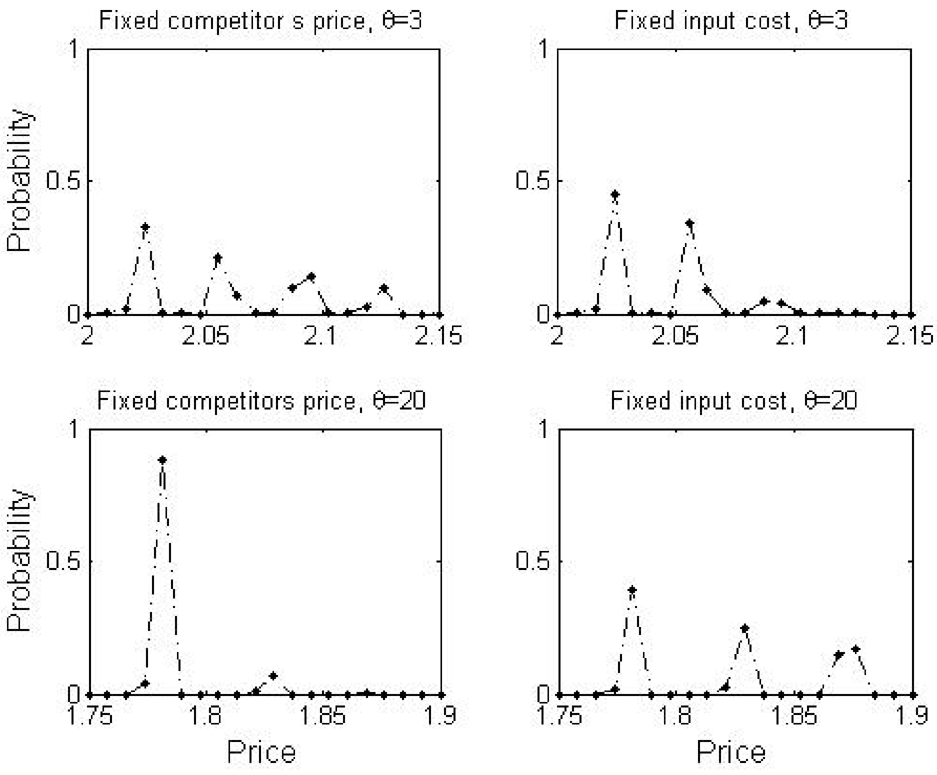
Let us study a model of a seller allocating his attention between unit input cost of his product and competitor's price. The seller faces a consumer, who has nominal endowment e = 1, she desires to consume two different products, whose prices are p1 and p2, to maximize the CES utility aggregate

$$U = \left(c_1^r + c_2^r\right)^{1/r}, \tag{1}$$

subject to a budget constraint: $c_1 p_1 + c_2 p_2 \leq e$. $\theta = 1/(1 - r)$ is the elasticity of substitution, $\theta \in (1, \infty)$. The consumer is assumed to have unlimited abilities to process information about the two prices, her demand for the product 1 is:

$$c_1(p_1, p_2) = \frac{1}{p_2\left(p_1/p_2 + (p_1/p_2)^{\theta}\right)}. \tag{2}$$

Figure 1: Distribution of price



A seller of the product 1 maximizes the expectation of his profit,

$$\Pi(p_1, p_2, \mu) = c_1(p_1, p_2)(p_1 - \mu). \tag{3}$$

He first processes information about the competitor's price, p_2, and his unit input cost, μ. Finally, he selects his own price, p_1. For the purposes of this example, we study a partial equilibrium only, taking distributions of p_2 and μ as given. Let μ be uniformly distributed in (1, 1.1) and p_2 uniformly in (2, 2.2). We need to solve (11) – (15) in Appendix. A source variable is a vector (p_2, μ), while the response variable is the seller's price, p_1.

Figure 1 summarizes responsiveness of the seller's price to unit input cost and to competitor's price, both in less and more competitive markets. When $\theta = 3$, i.e. goods are relatively poor substitutes, price p_1 varies relatively more with changes in input cost than with shocks to competitor's price. A distribution of prices for flexible input cost and a fixed competitors price (upper left graph) is more spread out than when input cost is fixed and competitor's price is varied (upper right).

On the other hand, when $\theta = 20$, i.e. goods are much better substitutes - the market is more competitive, price p1 responds more or less to changes in the competitors price only.

When $\theta = 3$, the seller possesses tighter knowledge about μ, while if $\theta = 20$, then almost all information capacity is spent on tracking p2.

1.2 Choice of Information Amount: Competitive Industries Generate Flexible Prices

In the first example, we addressed the question of attention distribution, while the total amount of information was kept fixed. However, we could assume that agents also choose how much information to process. Let RIκ stand for the original model of a rationally inattentive seller with a fixed information and RIλ denote a model with fixed unit cost of information. In RIλ, sellers find the processing somewhat unpleasant and process more information only as long as its cost is lower than the marginal benefits from it. Agents maximize expectation of profit,

$$\tilde{\Pi}(\mu, p, \kappa) = p^{-\theta}(p - \mu) - \lambda\kappa, \tag{4}$$

where λ is the cost of processing 1 bit of information about μ, and κ is the amount of information the price-setter chooses to process.

It turns out that sellers in highly competitive industries decide to process more information about input cost, because their profits are more sensitive to suboptimal pricing. We compare sellers of the same size that face demands with different elasticities. Elasticity of demand is a measure of the degree of competition in an industry. Magnitude of demand (size of the firm) determines shadow price of information and thus influences choices of how much information to process. Larger sellers decide to process more information. Normalizing the magnitude thus allows for unbiased comparisons of information choices across markets.

The seller maximizes

$$\tilde{\Pi}(\mu, p, \kappa) = \left(\frac{p}{p_{opt}(\mu_0)}\right)^{-\theta}(p - \mu) - \lambda\kappa. \tag{5}$$

Figure 2 shows computational results of the dependence of a selected κ on demand elasticity, θ, for unit input cost uniformly distributed in (0.8, 1.2) and $\lambda = 0.5 \cdot 10–3$. Optimal information capacity is an increasing function of θ.

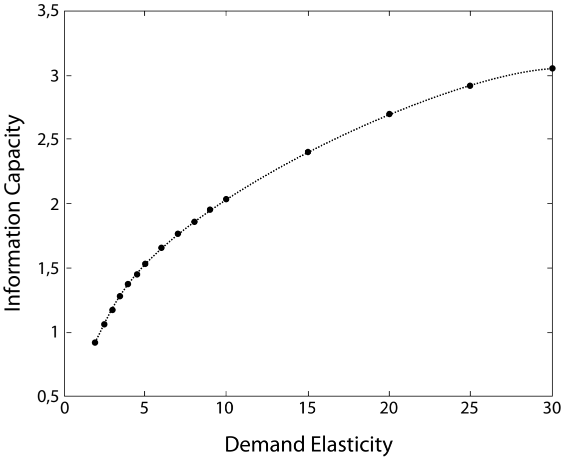
To justify this result analytically, we can apply the approach developed in Matějka (2010a). The corresponding approximate loss factor at μ_0 is

$$L(\mu_0, \theta) = \frac{\theta}{2\mu_0}. \tag{6}$$

It is a decreasing function of θ. The higher elasticity of demand, e.g. degree of competition, the bigger the loss from imperfect information about the input cost. Given the same levels of input cost and the same firms' sizes, agents process more information in more competitive industries. Moreover, since the price is tracked more closely, more information leads to more flexible prices.

Mutual information between two random variables is a measure of how much about one variable can be inferred from learning about the other. Even for an outside observer, the seller's higher information capacity implies that prices carry more information about the input cost.

Figure 2: Comparisons across industries



This finding relates to Hayek's famous defense of free markets, Hayek (1945), specifically on the grounds of markets' ability to convey information. Rational inattention implies that the more competitive a market is the more information can be extracted by observing its prices.

1.3 Dynamics and Aggregate Trade-Offs

In this section, we will use a model introduced in Matějka (2010a). It shows that unlike sticky-price models, rational inattention can generate frequent price changes together with delayed aggregate responses. The model's results agree very well with the empirical findings in Eichenbaum, Jaimovich, and Rebelo (2008).

In the following model, there are two stochastic variables to which the seller responds. Let the unit input cost be composed of two parts: an i.i.d. real unit input cost denoted by

μ and the second one be a serially correlated nominal variable A. μ is supposed to be an idiosyncratic volatile part of the input cost specific to the seller, while A plays the role of a slowly moving aggregate variable, e.g. a price level. The profit function takes the following form.

$$\Pi(A,\mu,p) = p^{-\theta}(p - A\mu). \tag{7}$$

A is a price index shifting the distribution of the nominal input cost $A\mu$. The aggregate variable takes two different values only and that it is Markov. Let the Markov process be symmetric with a probability of transition to the other state equal to t. A is binary, its distribution is determined by the probability of either one of the two states. Let the state variable be $x = Prob(A_L)$, where A_L stands for the lower value of A. The model's equations are formulated in Appendix.

For computations, I used $\kappa = 1$, $\theta = 3$, μ uniformly distributed over (0.8, 1.2), $A_L = 1$, $A_H = 1.1$, $t = 0.002$ and $\beta = 0.9992$. One period is supposed to be one week. $t = 0.002$ implies that the probability of changing a state (a 10% shock to the aggregate variable) at least once during a year is about 10%. The annual discount factor is 0.96.

Figure 3 shows the results of simulations over 120 periods. There is a shock to A in the period denoted as 1, when A switches from A_L to A_H. The top series in the figure presents one realization of a price series and the second one shows a time-series of knowledge about A in the same simulation.

The price setter processes information about $A\mu$ and responds to it, trying to target the optimal price $\theta/(\theta - 1)A\mu$. Although $A\mu$ is distributed over a continuous range in every period, prices again exhibit lots of rigidity of the values as well as in the i.i.d. case. Given prior knowledge about A, together with its true value, the distribution of prices as responses to realizations of μ is discrete. However, when knowledge about A changes, the distribution of prices changes too. For the used values of parameters (κ, t, A_H, etc.), knowledge adjustment is rather abrupt. The second series in Figure 3 shows a knowledge adjustment that is quite typical for all realizations of single simulations with these parameters. What varies from one simulation to another is the period in which the seller finds out that A has probably switched to a new value. The sudden change of knowledge is, however, not inherent to all solutions under rational inattention. The next subsection discusses this point in a little more detail.

The bottom two series in Figure 3 are prices and knowledge averaged over 10 000 runs. The average knowledge about A shifts slowly, while the average price does actually change abruptly in period 1. The variable of interest to the seller is in fact $A\mu$, not values of A and μ separately. Due to different dynamical properties of A and μ, and a non-uniform prior on $A\mu$, the agent does not process information exactly about $A\mu$ only. Although, the seller does pay special attention to $A\mu$, he also refines knowledge about other regions in the whole $A \times \mu$ space. In period one, after a positive shock to A, the seller is likely to find out that the value of $A\mu$ is high and thus the probability that a distribution's top price is realized increases. Due to the prior knowledge that A is probably at the lower state, the agent underestimates the true value of $A\mu$. Expected price adjusts abruptly, but still less than optimally. Since A stays at the higher level, the agent obtains signals on a high $A\mu$ several periods in a row and slowly learns that it is not due to a streak of high μ, but rather due to a jump in A. The average price further increases towards the new optimal level. Prices change frequently, but responses to shocks to the aggregate variable are delayed.

The difference between RIκ and RI_λ versions of the dynamic model correspond to differences between their static counterparts. Stochastic properties of the variables of interest (μ and A) influence the agent's choice of what pieces of information and potentially how much information to process. Let us vary these properties and study their implications for responses to shocks to the aggregate variable A.

1.3.1 Idiosyncratic Volatility

Thus far, μ was uniformly distributed in (0.8, 1.2). Table 1 summarizes numerical results for RIκ with κ = 1, θ = 3 and t = 0.002 for three different widths of the distribution of μ:

Figure 3: Two stochastic variables, sudden learning, t = 0.002, κ = 1

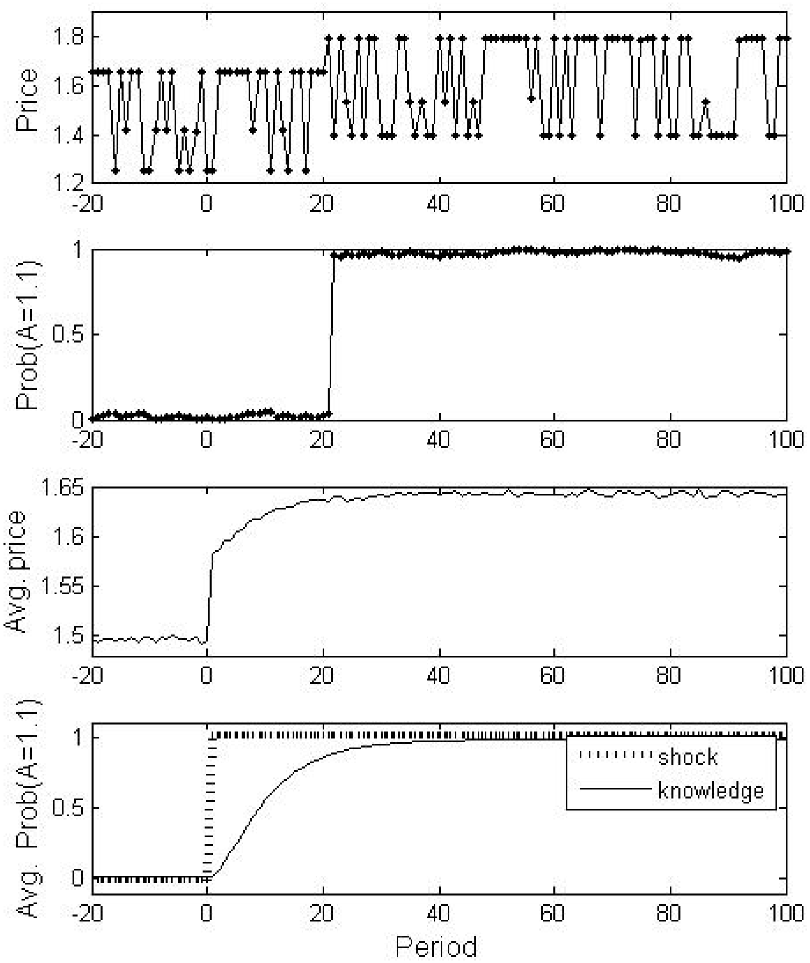
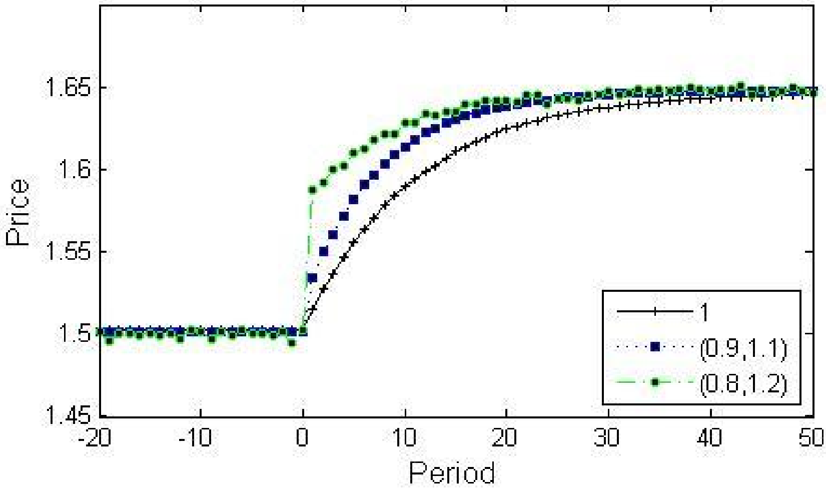


Figure 4: Average price response, 3 distributions of μ, λ = 0.003



for μ fixed at 1, uniformly distributed in (0.9, 1.1) and in (0.8, 1.2). Two characteristics of responses to a shock to A averaged over 10 000 runs are in columns 2 and 3. "1st per. adj." represents a portion of the average long-term adjustment that was realized during the first period, while "90% adjustment" denotes the number of periods it takes the average price until 90% of the full adjustment is realized.

Table 1: Implications of idiosyncratic volatility for average responses, κ = 1

μ	1st per. adj.	90% adjustment	profit loss
1	100%	1	0%
(0.9,1.1)	83%	2	0.28%
(0.8,1.2)	61%	11	1.09%

The more volatile is the seller's idiosyncratic part of the input cost the slower he responds to aggregate shocks. When μ is fixed at 1, 1 bit of information is sufficient to track innovations of the binary variable A perfectly. The column "profit loss" presents seller's losses in comparison with pricing under perfect information - this quantity was evaluated with both μ and A simulated according to their stochastic properties. As expected for RIκ, losses are higher in more volatile environments.

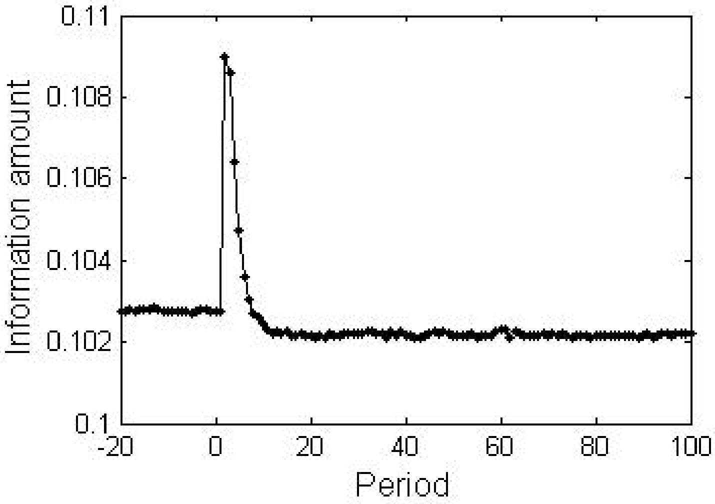
Results of the similar experiments for RI_λ, λ = 0.003, are shown in Table 2 and in Figure 4. Unlike RIκ, RI_λ generates faster responses to aggregate shocks when μ is more volatile.

Table 2: Implications of idiosyncratic volatility for average responses, λ = 0.003

μ	1st per. adj.	90% adjustment	profit loss	mean κ
1	8%	25	0.08	0.0086
(0.9,1.1)	22%	17	1.34%	0.014
(0.8,1.2)	60%	12	1.34%	0.86

The table also presents average κ that was selected by the seller during the simulations. Agents in RI_λ choose to process more information when volatility of input cost increases,

Figure 5: Average processed information, $\mu \in (0.9, 1.1)$, $\lambda = 0.003$, $t = 0.02$



which increases the marginal value of information. Since a rationally inattentive agent processes optimal joint signals about $A \times \mu$, then more total information also implies moreinformation about A. However, faster average responses do not always mean more precise responses - profit loss is the same for $\mu \in (0.9, 1.1)$ and $\mu \in (0.8, 1.2)$. The loss dropsdramatically when μ is fixed at 1. Input cost becomes a binary variable, therefore, the intuition derived from the linear-quadratic approximation does not apply very well.

In RI_λ , the amount of processed information is not constant. It varies according to the expected value of information given a prior. Figure 5 presents average information amount as a function of time. Immediately after the shock occurs, a large fraction of sellers realize there might have been a shock and they choose to process more. Later on, average selected capacity decreases. Once the transition period is over, the new equilibrium information capacity is actually below the initial level - the new average input cost is higher and the marginal value of information is thus lower.

1.3.2 Aggregate Volatility

Aggregate volatility can be adjusted by varying the Markov parameter t, the probability of transition between the two states. Tables 3 and 4, present characteristics of average responses for $\kappa = 1$ and $\lambda = 0.003$ for four different levels of t.

Table 3: Implications of aggregate volatility for average responses, $\kappa = 1$

t	1st per. adj.	90% adjustment	profit loss
0.001	56%	18	1.08%
0.002	61%	11	1.09%
0.006	73%	7	1.13%
0.02	80%	4	1.20%

Table 4: Implications of aggregate volatility for average responses, $\lambda = 0.003$

t	1st per. adj.	90% adjustment	profit loss	mean κ
0.001	55%	19	1.34%	0.86
0.002	60%	12	1.34%	0.86
0.006	73%	5	1.35%	0.88
0.02	83%	3	1.35%	0.92

More volatile A generates faster responses to its innovations in both of the models, $RI\kappa$ and RI_λ. This is due to a higher marginal value of processing new information about A if A is more likely to vary and due to the signal extraction of A from Aμ. When volatility of A increases, shocks to Aμ are more likely to be attributed to A. However, unlike in the Lucas' signal extraction of the whole Aμ, profit losses are higher when the aggregate environment is more volatile. Less stable Aμ is more difficult to be tracked precisely.

The effect of accelerated average adjustment is slightly stronger in RI_λ, since higher volatility provides additional motive for processing more total information.

Volatile aggregate environment generates higher losses and swifter responses and thus potentially also a steeper Phillips curve.

1.4 Nominal Rigidity Driven by Consumers' Inattention

In the previous sections, one has to use quite low information capacity, κ, or high cost of information, λ, to be able to generate quantitatively realistic rigidities. In many cases, low information capacity is not unappealing, consider a local businessmen selling homemade honey, but corporations such as Microsoft, IBM or GM also set prices rigidly, while probably having very good information about economic aggregates. In these cases, it is perhaps less likely that nominal rigidities would be driven by information constraints of the price-setters. This section shows that consumers' inattention suffices to generate the rigidity.

Consumers often do not realize what a product's exact price is at the moment of a purchase decision. This is inspired by the observation that some consumers just grab certain products in a supermarket without even looking at their prices. Many of us at least read price's first few digits, while ignoring the cents. Typically, we implicitly assume that prices end with .95 or .99. If the number of cents is actually 85, we may not spot it and still keep our initial guess. Sometimes, we read just the first digit only or none at all.

If it is unpleasant, i.e. costly, to inspect prices and if uncertainty about the true price can discourage consumption, then sellers could try to accommodate consumers with more predictable prices. It might be optimal for the seller not to respond to every minor change of input cost. Such frequent price changes would require consumers to pay lots of attention to the price, and if they did not want to, then they could rather decide to consume less.

Rationally inattentive agents learn about new innovations slowly. If there is a shock to an aggregate variable and if the seller's input cost is correlated with this variable, then the seller chooses to respond to such a shock gradually - he chooses to price in line with the consumer's expectations.

Imagine a consumer has some partial knowledge about shocks to energy prices. She also knows that energy prices are the main determinant of the input cost for her favorite local sauna club. The consumer's expectations about the admission prices to the sauna vary with what she knows about the current prices of energy. The sauna owner might postpone new price changes until consumers expect them to occur.

1.4.1 Model

The model has these features:

i) The input cost is drawn from a binary distribution in the period 0.
ii) The consumer's knowledge of the seller's cost evolves independently of the seller's actions. Knowledge is gradually refined.
iii) The seller's price is a function of the unit input cost and the time elapsed from the initial shock.

Let the seller's unit input cost be equal to an aggregate variable A. The seller is small and has a negligible influence on the consumer's knowledge about shocks to A - the knowledge evolves independently from the seller's pricing responses to A. A is drawn from asymmetric binary distribution $\{A_L, A_H\}$ in the period 0 and stays constant forever after. Consumers know that one of the two possible shocks is realized, but need to process information to find out which one it is. Such a setting with a one-time shock is both simpler to solve and yet illustrative enough to document the implications of gradual knowledge adjustment.

In fully-fledged models under rational inattention, we would specify consumers' preferences and allow them to choose what pieces of information to process. I will, however, assume one specific form of information structure. The qualitative properties of the results do not rest on this assumption.

Let us assume that the consumer's knowledge in period t has the same form as if the consumer acquired one signal through a binary channel with a noise level X (t). X (t) is decreasing in t, which models knowledge refinement[2]. With increasing time, there is

2 *No sequence of signals across periods is considered, just one signal, which gets tighter in latter periods.*

a higher probability that agents receive the correct signal. Posterior knowledge is thus more concentrated.

If $A = A_H$, then the probability that an agent receives the correct signal, $(A = A_H)$, is $1 - X(t)$. The posterior knowledge of an agent having received such a signal is $\{P(A_L) = X(t), P(A_H) = 1 - X(t)\}$. The posterior knowledge of agents who received the corrupted signal, $(A = A_L)$, is $\{P(A_L) = 1 - X(t), P(A_H) = X(t)\}$.

The seller chooses his pricing strategy. Unlike in the earlier sections of this paper, the strategy is not a function of the input cost only. The consumer's knowledge evolves even after period 0, when the input cost is kept fixed. Different consumer's knowledge can imply a different optimal pricing response to the same input cost. The pricing strategy takes the form $p = \tilde{p}(A, \{g(A)\}) = \tilde{p}(A, t)$, where $\{g(A)\}$ is the distribution of knowledge[3] in the population of consumers. $\{g(A)\}$ is determined by $X(t)$, which is pinned down by time t. The strategy can be expressed using two functions, $\tilde{p}_L(t)$ and $\tilde{p}_H(t)$, each corresponding to one level of unit input cost:

$$p = \begin{cases} \tilde{p}_L(t) & \text{if } A = A_L, \\ \tilde{p}_H(t) & \text{if } A = A_H. \end{cases} \tag{8}$$

Consumers are rational, they know the form of $\tilde{p}_L(t)$ and $\tilde{p}_H(t)$. Together with their knowledge about the aggregate shock (A determines which one of $\tilde{p}_L(t)$ and $\tilde{p}_L(t)$ is to be applied), the pricing strategy generates the consumer's prior on price. More specifically, the pricing strategy forms the prior's support, while knowledge about A determines the relative probabilities of its two points. The term prior reflects knowledge "before" a consumer processes information about the seller's price, but it is "after" she has processed information about the aggregate shock. Since the consumer's knowledge about A is independent of the seller's actions, optimal prices at different levels of $X(t)$ can be set independently of each other.

Definition 1. Model: $X(t)$ is given for all $t \in \{0.\infty\}$; it is a non-increasing function. For each t, the seller chooses $\tilde{p}_L(t)$ and $\tilde{p}_H(t)$, maximizing the expectation of his profit

$$\{\tilde{p}_L(t), \tilde{p}_H(t)\} = \arg \max_{\{\tilde{p}^0_L(t), \tilde{p}^0_H(t)\}}$$

$$\frac{1}{2} \left(\tilde{p}^0_L(t) - A_L\right)$$

$$\left(1 - X(t)\right)E[C|t, A_L] + X(t)E[C|t, A_H]$$

$$+ \frac{1}{2} \left(\tilde{p}^0_H(t) - A_H\right)$$

$$X(t)E[C|t, A_L] + \left(1 - X(t)\right)E[C|t, A_H] \quad o. \tag{9}$$

3 *It is actually a distribution of distributions.*

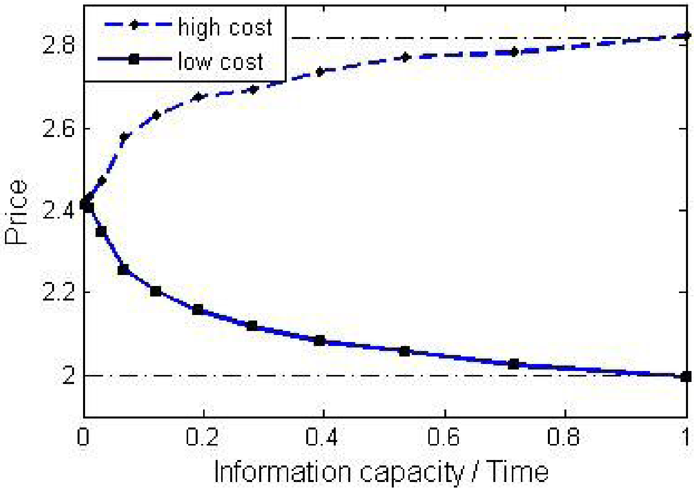
The expression for expected profit weights the true realizations of A_L and A_H and also the consumer's priors generated by receiving signals on A_L or A_H. $E[C|t, A_L]$ denotes the consumption expectation when a consumer's prior is determined by a signal pointing to AL with a noise level X (t) - the corresponding prior is $\{g(\tilde{p}_L(t)), g(\tilde{p}_L(t))\} = \{1 - X(t), X(t)\}$. On the other hand, a prior determining $E[C|t, A_H]$ is $\{1 - X(t), X(t)\}$.

For each X (t), the numerical representation of the optimal pricing strategy can be found simply by evaluating the expected profit for all combinations of $\{\tilde{p}_L(t), \tilde{p}_H(t)\}$. Let noise decrease at the following rate:

$$X(t) = 0.5 - 0.05t, \forall t \in \{0..10\}. \tag{10}$$

If the realized value is $A = A_H$, then the seller's price gradually increases until it reaches the full information price in period 10. Otherwise, it gradually decreases. For simplicity,

Figure 6: Gradual price adjustment, scaled to information amount



let κ = 0. Consumers do not process any additional information about the seller's price, they only use their knowledge about the aggregate variable. If κ > 0, the same optimal prices would correspond to higher levels of X.

Figure 6 shows the resulting solution. Consumers possess very little knowledge about A in early periods. They know the seller's pricing strategy, but have difficulties distinguishing between the two different values of prices, $\tilde{p}_L(t)$ and $\tilde{p}_H(t)$, that can be realized in the particular period. If X (t) = 0, consumers always acquire the correct signal, then the seller sets the perfect information optimal prices, which are represented by the dash-dotted bounds. If X (t) = 0.5, consumers can not tell at all which of the two prices was realized - in such a case, the seller chooses to set one price only. Like in the static model, consumers consume more when they are less uncertain about prices. With the increasing probability of the correct signal, optimal prices $\tilde{p}_L(t)$ and $\tilde{p}_H(t)$ are set further and further away from each other.

Figure 6 presents the impulse response of prices to an aggregate cost shock. Although the price-setter is perfectly attentive, prices adjust slowly. The more information the consumer processes, the faster the prices adjust. This implies that prices adjust faster if, for instance, the price makes up for a relatively large portion of the consumer's budget.

2 Implications and Conclusions

Most importantly, this paper shows that rational inattention can generate several properties of price dynamics that are observed in data. We find that prices can change quite frequently and yet generate inertial of nominal aggregates, Section 2.3. In data, Bils and Klenow (2002); Eichenbaum, Jaimovich, and Rebelo (2008), prices in retail stores stay fixed on average for less than 4 months. On the other hand, price level fully responds to an aggregate shock only after at least a year. The presented model reconciles with such findings. We thus conclude that rational inattention could provide the proper microfoundations for the models of nominal rigidities used in monetary policy.

Moreover, all of the following implications of the presented model agree with the evidence.

1) Prices are more flexible in volatile and competitive industries, Sections 2.3.1 and 2.3.1.
2) Prices are more flexible in volatile aggregate environments, Section 2.3.2.
3) Prices of small-budget products are less flexible, Section 2.4.

What does all this imply? For monetary policy, the findings have three main implications:

1. The proposed modeling approach based on rational information.
2. Monetary policy can become ineffective in stimulating output during crises. We find that prices become flexible when the aggregate environment is more volatile. In such cases, the price-setters choose to pay more attention to new shocks. Paying more attention then implies that prices adjust faster. If prices are flexible, then real effects of monetary policy diminish. Unfortunately, this can occur at exactly the times when the stimulation of output could be highly desirable.
3. Optimal monetary policy should focus on stabilizing the price level. The big question in monetary economics is how to balance trade-offs between stabilizing price-level and output. Recently, Paciello and Wiederholt (2011) find that when decision- makers in firms choose how much attention they devote to aggregate conditions, complete price stabilization is optimal also in response to shocks that cause inefficient fluctuations under perfect information. This finding goes in the opposite direction to what standard sticky-price models imply. Under sticky-prices, i.e. when explicit adjustment costs occur, pure price level stabilization is not optimal in case when mark-up shocks or taste shocks occur. Rational inattention thus provides additional reassurance that central banks having the price level stabilization as their primary objective, e.g. the Czech National Bank, choose the optimal policy objective.

The presented model provides some intuition for implications for fiscal policy too. The model we studied, and is formulated in Appendix, is a general setup of responses of inattentive agents to exogenous shocks. Those shocks can be of non-monetary nature too, while most results would still hold.

In 2008 and 2009, policy-makers around the world considered what actions to take to stimulate the output and employment as quickly as possible. Rational inattention implies that while the adjustment of the federal funds rate may be ineffective at turbulent times, fiscal policy can generate desirable results. This is exactly for the reason that at such time agents pay more attention, which weakens the effects of monetary policy, but can strengthen effects of fiscal policy.

Lower taxes go sometimes wasted when public does not notice them. During 2008-2009, public paid much more attention to business news than usually. This period was thus particularly receptive to the lowering of income taxes, including the social security payments. At the same time, higher consumption taxes were the prime candidate to fill the government budget. When noticed, higher consumption taxes increase inflation expectations, which only increases current consumption and thus output too.

On the other hand, increased government spending may be far from being the optimal action. It is true that effect of such spending would be the quickest, but it might be very weak. When agents know about the negative nature of the aggregate shock, then they are better aware of the future budget consequences of the current spending and may not increase their consumption. In other words, Ricardian equivalence becomes stronger during such volatile times.

The model presented in Section 2.4 and its extended version in Matějka (2010b) also provide a novel framework for quantification of costs of too complicated laws and tax codes. It has become common knowledge that complicated tax codes are detrimental. We, however, still do not know what the proper trade-off between the code's theoretical optimality and the optimal complexity that can be handled by real citizens. The presented model of the rationally inattentive consumer has the features needed to tackle the problem: the consumer (citizen) finds it complicated to understand all details of the pricing strategy (tax code), so the seller (government) chooses to keep the prices more rigid (simpler tax code) to accommodate the consumer.

Although, this is mainly a theoretical paper, the potential implications seem important enough to further study and develop the theory of rational inattention.

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Appendix: Formulation of Models under Rational Inattention

Response strategy under rational inattention. Let g(Y) be the agent's prior knowl-edge about Y , κ be her information capacity and U (y, z) be the indirect utility function. Her decision strategy f (Y, Z) is a solution to the following maximization problem.

$$f(Y, Z) = \arg\max_{f^0(\cdot,\cdot)} E[U(Y, Z)] = \arg\max_{f^0(\cdot,\cdot)} \int U(y, z) f^0(y, z) dydz, \tag{11}$$

subject to

$$\int f^0(z, y) dz = g(y) \quad \forall y \tag{12}$$

$$f^0(y, z) \geq 0, \quad \forall y, z \tag{13}$$

$$I(Y; Z) \leq \kappa, \tag{14}$$

$I(Y;Z)$ is mutual information between Y and Z, defined as

$$I(Y; Z) = H(Y) - H(Y|Z) = = \int f(y, z) \log \frac{f(y, z)}{g(y) f(z)} dydz. \tag{15}$$

(12) requires consistency with prior knowledge, agents can not process more information simply by forgetting what they knew in advance, and (13) states non-negativity of a prob-ability distribution. (14) is the information constraint.

The recursive formulation of the seller's dynamic problem, a RIκ version, is as follows.

$$V(x) = \max_f \int \Pi(A, \mu, p) + \beta V \mathbf{i} (x^0) f(A, \mu, p) dA d\mu dp, \tag{16}$$

subject to

$$x^0 = f(A_L|p)(1 - t) + 1 - f(A_L|p) t \quad Z \tag{17}$$

$$f(A, \mu, p) dp = g(A, \mu) = g_1(A) g_2(\mu) \tag{18}$$

$$g_1(A_L) = x \tag{19}$$

$$I(A, \mu; P) \leq \kappa. \tag{20}$$

f (A, μ, p) is a joint distribution summarizing the seller's choice of signals and responses in the given period, (17) is law of motion for knowledge, it generates a prior on A in the fol-lowing period from a posterior in the current period via a Markov process with the transi-tion probability t. (18) is the constraint on a prior, μ and A are assumed to be independent. $g_2(\mu)$ is fixed, $g_1(A_L) = x$,(19), which implies $g_1(A_H) = 1 - x$. (20) is the traditional constraint on mutual information between the source variables μ and A, and a response variable p.