Information Acquisition and Excessive Risk: Impact of Policy Rate and Market Volatility
Získávání informací a nadměrné riziko: role úrokových sazeb a volatility na trzích

VOLHA AUDZEI

Abstract
Excessive risk-taking of financial agents drew a lot of attention in the aftermath of the financial crisis. Low interest rates and subdued market volatility during the Great Moderation are sometimes blamed for stimulating risk-taking and leading to the recent financial crisis. In recent years, with many central banks around the world conducting the policy of low interest rates and mitigating market risks, it has been debatable whether this policy contributes to the building up of another credit boom. This paper addresses this issue by focusing on information acquisition by the financial agents. We build a theoretical model which captures excessive risk taking in response to changes in policy rate and market volatility. This excessive risk takes the form of an increased risk appetite of the agents, but also of decreased incentives to acquire information about risky assets. As a result, with market risk being reduced, agents tend to acquire more risk in their portfolios then they would with the higher market risk. The same forces increase portfolio risk when the safe interest rate is falling. The robustness of the results is considered with different learning rules.

Keywords
Rational Inattention, Interest Rates, Financial Crisis, Risk-taking

Abstrakt
Nadměrné podstupování rizika zástupci finančního trhu získalo po nedávné finanční krizi mnoho pozornosti. Nízké úrokové sazby a tlumená volatilita na trhu během období Velkého zklidnění (Great Moderation) jsou někdy obviňovány ze stimulace podstupování rizika, které vedlo k nedávné finanční krizi. V posledních letech, kdy centrální banky po celém světě provádí politiku nízkých úrokových sazeb, a zmírňují tržní rizika, je akutní otázka, zda tato politika nepřispívá k vytvoření další úvěrové konjunktury. Náš článek se zabývá tímto tématem z pohledu získávání informací zástupců finančního trhu. Sestavíme teoretický model, který zachycuje nadměrné podstupování rizika v reakci na změny úrokové sazby a/nebo tržní volatility. Toto nadměrné riziko získává formu zvýšené chuti zástupců finančního trhu riskovat, ale také snížené motivace získávat informace o rizikových aktivech. V důsledku sníženého tržního rizika, mají zástupci finančního trhu tendenci hromadit více rizika ve svých portfoliích než v případě s vysokým tržním rizikem. Stejné mechanismy zvyšují riziko portfolia, když je úroková sazba snížena. Robustnost získaných výsledků je posuzována z hlediska různých pravidel učení.

Klíčová slova
Rational Inattention, úrokové sazby, finanční krize, podstupování rizika

JEL Codes

E44, E52, G14, D84

Introduction

The paper is motivated by the debate about whether a low policy rate has contributed to the recent financial crisis and if the ongoing policy of low interest rates is contributing to the building up of a new financial bubble. There are voices among policy-makers and academics suggesting that one could observe worrying tendencies of risky asset accumulation[1]. There is evidence of an increased risk appetite, which is believed to be attributed to accommodative monetary policy conditions and subdued market volatility (for the evidence see, e.g., Bank for International Settlements 2014). At the same time both proponents and opponents of a low policy rate do not have clear answers as to what tools a central bank should use in order to maintain price stability and stimulate output growth on the one side, and financial stability on the other (for a recent debate on this see Stein 2013 and Bernanke 2013).

The question asked in this paper is if endogenous information acquisition can drive over-accumulation of risk when safe interest rates or market volatility is reduced. It is common that in portfolio choice models with rational expectations, investment into a risky asset is linear in excess return. In our model, when the policy rate or market volatility falls, risk accumulation in the economy increases in a nontrivial way.

We capture the excessive risk accumulation by modeling information decisions. Financial agents invest in information to reduce the variance of their forecasts. We show that when market volatility declines, agents invest into information less and acquire more of a risky asset. This results in an even larger portfolio risk than in the economy with higher market volatility. With interest rates being lowered, our model not only captures the standard "search-for-yield" effect, where financial intermediaries invest more into risky assets. We also show an increase in agents' ignorance about the asset quality. With low information investment and large risky asset holdings it implies a larger portfolio risk accumulation.

The main contribution of our model to the current debate is that it mimics excessive risk-taking of financial agents. We show that average risk monitoring declines with lower interest rates despite the growth in excess return on a risky asset. Another result is over-accumulation of risky assets in a low risk environment. That is to say with low variance of risky asset return, agents take more risk in their portfolio than they would have with a high risky asset variance. This effect is explained in our model with just one deviation from rational expectations: agents do not know the future return, but only its distribution, i.e. there is no assumption of agents' irrationality. In our model, this result is driven by a decline in risk monitoring in low risk environment. Combined with an increase in risky asset acquisition, it results in higher portfolio variance compared to high variance environment.

1 *For the evidence see Stein (2013); the recent examples of uncertainty among policy makers could be found in articles by Chris Giles "Central Bankers Say They Are Flying Blind " and "IMF warns on risks of excessive easing" in The Financial Times, April 17, 2013.*

To check the robustness of the results, in the spirit of Nieuwerburgh and Veldkamp (2010) we consider two alternative learning functions, a linear and an entropy based. The rise in portfolio risk when the safe interest rate falls is robust to a learning rule specification. The increase in risk with falling market volatility is more pronounced in a linear learning rule.

1 Related Literature

Our study relates to the several stands of literature. First, there is the literature on the role of interest rates in mitigating or stimulating asset booms, in particular papers providing empirical evidence that easier monetary policy is associated with higher risk-taking. Maddaloni (2011) concludes that, for the euro area and US, low short-term interest rates cause softening of the banks' lending standards. Additional support for a risk-taking channel of monetary policy can be found in Gambacorta (2009) and Ongena and Peydro (2011). Adrian et al. (2010) find empirical support for the notion that monetary policy effects the supply of credit, operating through the term spreads; and that monetary policy can influence risk appetite. Ahrend (2010) focuses on a different aspect of the financial imbalances - on excessive asset prices growth, and finds that low interest rates cause growth in some asset prices in OECD countries, particularly on the housing market. Detken and Smets (2004) come to the similar conclusion that low policy rates coincide with asset price booms. The evidence on the dynamic interaction between stock prices and Federal Reserve policy rate is provided by Laopodis (2010). White (2012) discusses the "unintended consequences" of easy monetary policy, among which are misallocation of credit and structural changes in the financial sector, e.g. movements from traditional banking model to shadow banking. Statistical evidence that a long period of low interest rate and low market volatility have contributed to excessive risk-taking is summarized in the Annual Report of the Bank for International Settlements (2014).

There are theoretical studies focusing on the channels through which monetary policy affects risk-taking or asset prices. Taylor (2007 and 2010) suggests that the Fed´s low rates stimulated a house price boom through credit growth. The several mechanisms through which the risk-taking channel of monetary policy could work are mentioned in Borio and Zhu (2008). In particular, search-for-yield implies that low interest rates result in a low return on the safe assets, which pushes investors to accumulate more of the risky ones in the search for an acceptable portfolio return. Also low interest rates imply a lower discount factor for evaluation of assets or income flows, causing higher risk tolerance. Our model incorporates both of these channels within the bank's portfolio choice problem.

The banks risk monitoring incentives in connection with monetary policy are studied in the model of Dell Ariccia et al. (2010). Their findings depend on the banks capital structure and the possibility of adjusting it. They conclude that with a flexible capital structure monetary policy easing leads to higher leverage and risk-taking. Their approach, however, is different from that pursued in this paper in several respects. They concentrate on a partial equilibrium model, where banks choose the probability of loan repayment subject to costs. Therefore, in their model banks do not learn about the asset quality, but invest to increase return probability. We build a general equilibrium model where banks are uncertain about the risky asset return, but might invest in reducing their uncertainty. That is, learning does not influence the return probability, but makes banks more informed. Therefore, we capture two aspects of risky behavior - investment in an asset known to be risky and investment into learning about the asset quality.

Another strand of literature our study is related to is dedicated to the learning and expectation formation and relaxation of the assumption of rational expectations. Among the papers to support the importance of imperfect expectations and learning are Boz and Mendoza (2010), Bullard et al. (2010), Kurz and Motolese (2010), Lorenzoni (2009), Adam and Marcet (2010). Empirical support for the role of imperfect expectations can be found in Fuhrer (2011) and Beaudry et al. (2011). In this paper we incorporate the idea that agents do not have perfect foresight and have to form subjective expectations about risky asset return. We use the approach of Nieuwerburgh and Veldkamp (2010) to model the banks decisions to invest in learning about the risky asset. In Nieuwerburgh and Veldkamp (2010), the investor draws an additional signal about asset return, and pays for an increase in the signal precision before observing it. We modify their formulation for information acquisition, so that in our model agents select the information budget depending on risk premia and market volatility.

To conclude, our study is motivated by rich empirical evidence. Our model explores causalities between monetary policy and agents' risk-taking. We also show that prolonged periods of low interest rates or low risk lead to excessive accumulation of risk.

The remainder of the paper begins with analysis of a partial equilibrium model to describe the intuition for the main results. In section 3 the financial sector is described, and the intuition for excessive risk-taking is presented in section 4 within a partial equilibrium. In section 5 we complete the model for general equilibrium and then proceed with the calibration, simulations and discussion in section 6. The last section concludes.

2 The Model of Financial Sector

Consider a model with a financial intermediary, bank, a manufacturing firm and a household. The assets in the economy are manufacturer claims (a risky asset) and reserves (a safe asset). The risk in manufacturer claims comes from the uncertainty about future productivity. All the agents in the economy know the productivity distribution. The household puts savings in the bank (in the form of investment), and the bank transfers all its profit back to the household. The safe and risky interest rates are set by the market.

The bank is risk-averse, which is motivated by the fact that banks are often subject to regulations and have reputational concerns for the safety of their deposits. We then expand the model and grant financial intermediary access to a noisy signal about future productivity. This signal helps the agents to reduce the variance of their forecast. Yet they have to pay for it. Banks are Bayesian, they form forecasts of risky returns as a weighted average of their prior and the signal.

We abstract from any nominal variables in the model. All the prices and returns are real. In what follows, we present the model set-up. We start with a partial equilibrium model to illustrate the mechanism of the excessive risk-taking and information acquisition. Then we simulate general equilibrium model to study the model dynamic and potential role of interest rates feedback[2].

We start with a description of the financial sector.

Banks. The bank is risk-averse and has mean-variance utility in its next period net return:

$$\max_{k_t^b} E_t\Pi_{t+1} - \frac{1}{\rho} Var\left(\Pi_{t+1}\right) \tag{1}$$

where ρ is the risk aversion parameter, k_t^b is the bank's risky asset holdings and Π_{t+1} stands for the next period return. That is, portfolio variance is costly and the bank, therefore, has incentives to reduce it. The next period return consists of the return on the bank's portfolio minus the information budget:

$$\Pi_{t+1}^b = d_t R_t^s + k_t^b \left(R_{t+1}^r - R_t^s\right) - b_t \tag{2}$$

where d_t is household investment, R_t^r and R^s are respectively gross returns from risky and safe assets, b_t is the information budget selected by the bank. The bank's future return depends on the amount of funds it has for investment - d_t, and from a composition of its portfolio - quantity of risky asset, k_t^b: Note that the return is reduced by the information investment, b_t:

The bank's objective is to maximize (1), and the choice variables are information budget, b_t, and risky asset quantity k_t^b. Compared to the strand of literature on rational inattention with exogenous capacity constraint, here we endogenize capacity and formulate it in budget terms.

Maximizing the bank's utility, we get its holdings of the risky asset:

$$k_t^b = \frac{E_t R_{t+1}^r - R_t^s}{\rho \hat{\sigma}_t^2} \tag{3}$$

where $\hat{\sigma}_t^2$ is risky asset return variance. Sign '^' stands for posterior variance, updated after information decisions. As is typical in the literature, the amount of risky assets bought is increasing with excess return, $E_t R_{t+1}^r - R_t^s$, and is decreasing with risk aversion, ρ, and risky asset return variance $\hat{\sigma}_t^2$.

For simplicity, we make the bank transfer all its profit to the household in return to their savings, d_t.

Information Acquisition. The information acquisition is modeled similar to Nieuwerburgh and Veldkamp (2010). In their paper an investor is allocating his/her exogenously limited capacity to learn between different assets depending on his/her portfolio decisions. In our

2 *In our model a risky interest rate could be viewed as a reverse of the asset price. With larger demand for a risky asset, it drops, potentially offsetting higher risk appetite.*

model, we endogenize learning capacity by replacing it with the budget, b_t. The bank then chooses the budget to determine how much to learn subject to fixed learning costs, *a*.

Financial intermediaries can reduce the variance of their return forecast by investing into additional signal and pay costs proportional to the variance reduced. The decision to monitor is taken ex-ante signal realization. For this purpose, the period is decomposed into sub-periods. The timing is as in table 1.

Table 1: The Timeline of Information Decisions

subperiod 1	**subperiod 2**
$\mu_t \sim N(R^r_{t+1}, \sigma^2_t)$	information signals are realized
expected posterior return is $E\hat{\mu} \sim N(\mu, \hat{\sigma}^2_t)$	$\hat{\mu}$ is formed using Bayes rule,
budget, b_t and $\hat{\sigma}^2_t$ are chosen	and portfolio is chosen: k^b_t

In table 1 μ_t is the bank's prior about future return, R^r_{t+1}, $E\hat{\mu}$ is the posterior the bank expects to get after observing the signal. $\hat{\sigma}^2_t$ is the posterior variance after observing the signal[3].

In the first subperiod the agent has prior variance, $\hat{\sigma}^2_t$; and expected return, μ_t, both coinciding with true moments of return distribution. The agent decides what budget to allocate to information decision. The choice of the budget determines by how much the variance will be reduced. In the spirit of Nieuwerburgh and Veldkamp (2010) we interpret it as an investment into purchasing additional market data, when an agent does not have prior knowledge of what is in the data, but knows that this data will sharpen his/her forecast. We model this decision as a choice of budget that determines posterior variance, $\hat{\sigma}^2_t$. When choosing the budget and posterior variance, agent takes into account what the return expectations will be after the signal is observed. In other words, the agent has to form expectations about return expectations: expected posterior $E\hat{\mu}$. Yet before paying for the signal and observing it, the expected posterior equals the prior $E\hat{\mu} = \mu$.

When taking decisions in subperiod 1, the agent rationally anticipates the demand for the risky asset in the subperiod 2 as in (3) where $\hat{\sigma}^2_t$ is posterior variance of the return. Thus, with the information investment - budget bt and (3), the banks utility is rewritten:

$$\max_{b_t, k^b_t} E_t \Pi_{t+1} - \frac{1}{\rho} Var\left(\Pi_{t+1}\right) \tag{4}$$

subject to the learning rule:

$$f\left(\sigma^2_t, \hat{\sigma}^2_t\right) \cdot a \leq b_t, \tag{5}$$

3 *All posterior variables are formed using Bayes rule.*

and non-forgeting constraint: $\sigma_t^2 - \hat{\sigma}_t^2 > 0$. a is cost of reducing the variance, and $f(\sigma_t^2, \hat{\sigma}_t^2)$ is the learning function. The function is continuous and monotone in both of its arguments, it is increasing in initial variance, σ_t^2, and is decreasing in posterior, $\hat{\sigma}_t^2$. Intuitively, the more we reduce the posterior variance relative to the prior, the more we should pay. We assume that the information budget is exhausted so that (5) becomes equality. Then with the properties of our learning function, the choice of the information budget, b_t, uniquely determines the posterior variance and captures the information decision of the bank.

In the following section we consider risk-taking decisions of the bank in a partial equilibrium to identify risk driving forces.

Aggregating Financial Markets. The total investment into the safe asset, res, is given by the bank's financial resources not invested into the risky asset:

$$res_t = d_t - k_t^b$$

The investment into the safe asset is determined as deposits, dt, that was not invested in the risky asset, k_t^b.

Recall, that the risky asset in the model is the investment in the manufacturing firm, which uses it to build new capital. The manufacturing firm does not have funds for investment on its own. To invest it has to sell its claims to the bank. Thus, the total investment into the capital is then given by the bank's risky asset holdings:

$$I_t = k_t^b$$

3 Excessive Risk-Taking and Information Acquisition

In this section we analyze the two channels through which a bank accumulates risk in the portfolio when the safe interest rate is reduced or market volatility declines. One of them is clear from (3): whenever the safe interest rate drops, it increases the risk premium and makes the risky asset more attractive. Similarly, when asset variance is reduced, the bank rationally increases holdings of the risky asset. The other channel highlighted in this paper is a change in information acquisition: reduction in the information budget. Through this channel, the bank increases the riskiness of the asset per se by choosing to learn less about it. The portfolio risk then, as a product of risky asset holdings and return variance, increases with the lower interest rate and, in some cases, lower market volatility.

At first glance, the reduction in information acquisition with increase in risky asset holdings might seem counter-intuitive. It could be suggested that with larger asset holdings, agents would like to learn more about them. For example Nieuwerburgh and Veldkamp (2010) found that when allocating fixed learning capacity between the assets, agents allocate more to those assets they invest more into. Here, we should remind the reader, that in our paper we are studying not the allocation of the fixed capacity, but the determination of this capacity: by how much agents are willing to reduce their expected income in order to reduce the income variance. Also this capacity, in the form of the information budget, is itself a function of expected return and initial variance. It describes a trade-off between the return the agent expects to get and variance he/she would like to reduce. Below, we study the properties of the information budget for specified learning functions.

As learning function choice could influence the results (and we show later that this is the case), we consider alternative functions. Nieuwerburgh and Veldkamp (2010) show that the choice of utility function and learning technologies in uences results quantitatively and, sometimes, qualitatively. They consider mean-variance and exponential utility functions, and three learning rules: one linear and two entropy based measures. Below, we study mean-variance utility under linear and entropy learning functions.

Information Budget and Comparative Statics. As in Nieuwerburgh and Veldkamp (2010) we consider alternative learning functions, $f(\sigma_t^2, \hat{\sigma}_t^2)$ in (5): a linear rule and an entropy based. The linear function implies that the bank pays fixed costs, a, for each unit of the linear decline in the variance:

$$b_t = a(\sigma_t^2 - \hat{\sigma}_t^2) \tag{6}$$

Linear constraint is an intuitive rule and simple to work with. The one caveat is that it is marginally as costly for the agents to reduce the variance by 1% as by 100%. Agents potentially could choose to learn the whole truth and choose the posterior to be zero. This, of course, is very costly for them in absolute terms of linear costs, a, and this never happened in our simulations. But in the general case, one should consider this possibility.

The entropy based constraint implies that the agent pays for each unit of log variance decrease. One can find some variation in the definition of the entropy based learning rule. For example, in Nieuwerburgh and Veldkamp (2010) it is the simple ratio of prior to posterior variance. Mackowiak and Wiederholt (2009) use the logarithm of base 2, while there are many papers on rational inattention using a natural logarithm (e.g. Matejka and McKay (2015) and Cabrales et al. (2013)). In our definition of entropy we follow Mackowiak and Wiederholt (2009)[4]:

$$b_t = a \cdot log_2\left(\frac{\sigma_t^2}{\hat{\sigma}_t^2}\right) \tag{7}$$

The advantage of the entropy rule is that when the agent gets closer to learning the true state of the world (posterior variance goes to zero), the required budget goes to infinity. The entropy constraint is also well-motivated for analysis of processing the information subject to limited capacity. In our case, however, the agent's decision resembles more a choice of a quality of market report to buy or market expert to pay, than processing market data him/herself. That is, in our view, both types of constraints are well reasoned here.

To select the information budget the agent maximizes the utility as in (4), but the decision is now divided in two subperiods. The information budget is chosen in the first subperiod:

$$\max_{b_t} E_{t,1}\left(E_{t,2}\Pi_{t+1} - \frac{1}{\rho}Var_{t,2}(\Pi_{t+1})\right) \tag{8}$$

subject to (3) and posterior variance, $\hat{\sigma}_t^2$, given by one of the learning rules: (6) or (7).

4 *The results with a natural algorithm do not differ qualitatively, and there is a minor quantitative difference.*

Note, that in (8) the agent chooses b_t in the first subperiod before knowing his expected return in the second subperiod (before the signal - market report - is realized). Adopting the formula from Nieuwerburgh and Veldkamp (2010), formula 14, we have:

$$\max_{b_t} -0.5 + \frac{\sigma_t^2}{\hat{\sigma}_t^2} \cdot \left(1 + \frac{(\mu_t - R_t^s)^2}{\sigma_t^2}\right) - b_t$$

It is instructive to analyze comparative statics of the resulting solutions. In the partial equilibrium model we take as given both assets returns, $E_t R_{t+1}^r$ and its mean, and R_t^s. It will be convenient then to consider model's response to change in expected risk premia, $E_t R_{t+1}^r - R_t^s$. In a general equilibrium, both returns will be determined by the market clearing condition, with a stochastic component influencing risk asset return. In table 2, the changes in the information budget with respect to variables of interest are described (for full description of the derivatives, the reader is referred to the appendix).

Table 2: Comparative Statics: Information Budget

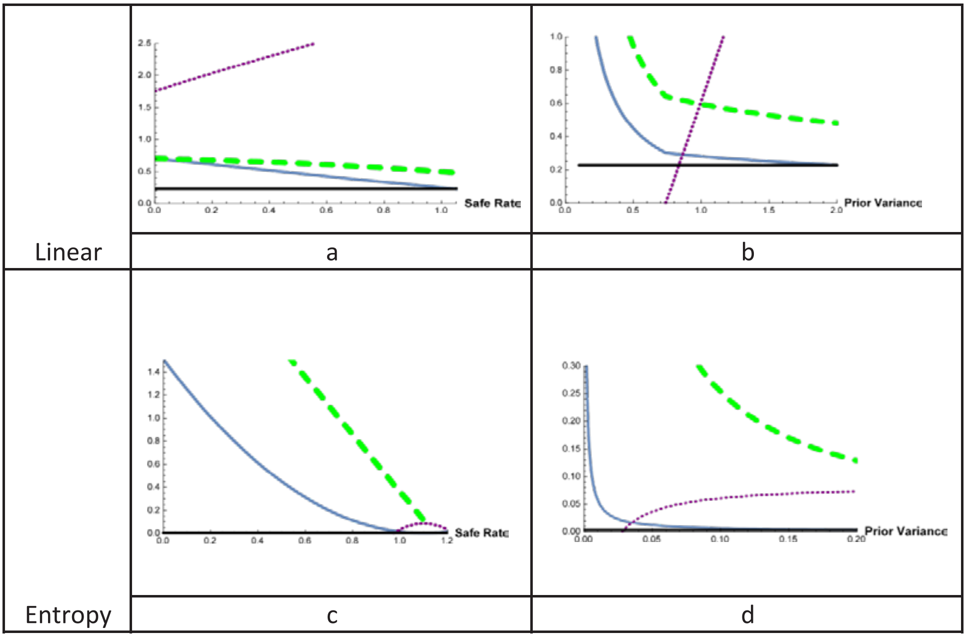
Information budget derivatives	Linear rule	Entropy rule
$\frac{\partial b_t}{\partial(\mu_t - R_t^s)}$	negative	negative for $\mu_t > R_t^s$, otherwise 0
$\frac{\partial b_t}{\partial \sigma_t^2}$	positive	positive for $\mu_t > R_t^s$, otherwise 0
$\frac{\partial b_t}{\partial a}$	positive	positive, but negative for relatively small

Comparing derivatives under both learning rules in table 2, we see the similar signs of the responses. The information budget rises when initial variance rises, so that with larger volatility in the market, agents are willing to sacrifice a larger budget to reduce uncertainty. Also, with a larger expected risk premium agents are willing to invest less in reducing the uncertainty, as the larger expected return compensates agents for taking a risk.

Table 2 explains the information channel of increase in risk-taking. When the safe interest rate falls, it decreases the expected risk premium (which is $(\mu_t - R_t^s)$), and decreases the information budget. With a lower information budget, the agent has a larger posterior variance. Similarly, with a lower initial volatility (prior variance), the agent decides to have a smaller information budget. The initial effect of a reduction in interest rate or initial variance on the risky asset position is positive. It could be suggested, that a small information budget and larger posterior variance may offset this effect. We show below that this is not the case in our model. The bank's risky position rises, and, together with small information acquisition, drives up portfolio variance.

Risk Accumulation in Partial Equilibrium. Calculating derivatives with respect to risk premium and prior variance, we find that risky asset holdings decrease in initial variance and increase in risk premium[5]. Figure 1 illustrates this point. The graphs were drawn with fixed interest rates. Later in the paper we analyze a general equilibrium model where interest rates are set by the market.

Figure 1: Risk Accumulation in a Partial Equilibrium



Note: dotted line corresponds to information budget b, dashed line - to risky asset holdings kb, solid line - to portfolio variance, bold solid line - steady state portfolio variance

In figure 1 panels *a* and *b* correspond to a model with a linear learning rule; and *c* and *d* to an entropy learning rule. The solid black line on all the graphs shows the initial (before reduction in safe interest rate and variance) portfolio variance. The solid blue line represents portfolio variance, its rise over the initial level shows the increase in portfolio variance. The channels of portfolio variance increase are clear from the figure: there is a decline in information acquisition, b_t[6], and an increase in risky asset holdings, k_t^b.

Panels *a* and *c* in figure 1 show, that when the safe interest rate falls, there is a larger risk accumulated in the portfolio. The risky asset position increases and the information budget falls. This resembles the debate that a low interest rate environment stimulated excessive risk-

5 *With the entropy learning, the risky asset position increases in risk premium for large enough* σ_t^2. *All derivatives are in the appendix.*

6 *At some point (panels b-d) the information budget hits zero. At this point, the model behaves the same as the one without information acquisition. Below this point, a sharper increase in risky asset holdings,* k_t^b, *is observed.*

taking during the Great Moderation. In our model, we capture also lower incentives to get information about the risky asset the agent becomes more ignorant about the asset quality.

A similar result is found for reduction in market volatility in panels b and d. Surprisingly, when the prior variance falls, the agent ends up with a larger portfolio risk than in a higher variance environment. This result is, again, driven by the information channel: an agent is willing to pay less for variance reduction when it is already small; and by larger risky asset accumulation when the risk gets smaller. This finding could be also be applied to the Great Moderation period, when market volatility was perceived to be low and financial agents demonstrated a higher risk appetite.

Of course, when trying to explain overaccumulation of risk during the Great Moderation, other forces besides the low volatility, mentioned, and a low safe interest rate environment could be considered. We show in this paper, however, that market volatility and low policy rates could be contributing factors to increase in risk preferences. These are also important factors to consider when addressing current central banks' policy of low interest rates and suppressing market volatility.

Next, we complete the model and consider risk accumulation in a general equilibrium.

4 General Equilibrium Model

Here we briefly describe the rest of the model and general equilibrium. Then we consider the equilibrium impact of the interest rate change on risk preferences and information acquisition, when there is feedback between the agents' asset holdings and market interest rates.

Household. There is a representative household which maximizes the following utility function:

$$\max_{\{c_{t+i},d_{t+i}\}_{i=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^i u\left(c_{t+i}\right) \quad (9)$$

subject to a budget constraint:

$$d_t + c_t = \pi_t^{fin} + \pi_t^{p} - t_t \quad (10)$$

where d_t is household savings, ${}^f\pi_t^{fin}$ is realized profit from the financial sector, π_t^p is realized profit from manufactures and t is tax. The household decides how much to consume and to invest in the bank. Its income is generated by the bank s and manufacturer's profits net of lump-sum taxes. $u(c)$ is twice differentiable and concave. Note, that we abstract from any labor decisions.

The consumption Euler equation looks standard and relates gross interest on savings to the stochastic discount factor:

$$u'(c_t) = R_{t+1}^d \beta E_t u'(c_{t+1}) \quad (11)$$

$$R_{t+1}^{d} = R_t^s + \frac{k_t^b}{d_t}(R_{t+1}^r - R_t^s) \quad (12)$$

Manufacturer. On the production side there is a representative producer with a production function:

$$y_{t+1} = z_{t+1}k_t$$

where z is stochastic productivity.

The producer needs to borrow money to finance investment (make new capital), and the law of motion for capital is then:

$$k_{t+1} = I_t + (1\text{-}\delta)k_t \quad (13)$$

The producer maximizes one period profit, which consists of revenues minus payment on the loan for investment purposes:

$$\max_{k_{t+1}} E_t \pi_{t+1}^p = E_t \left(y_{t+1} - R_{t+1}^r * I_t \right) \quad (14)$$

where R^r is the gross interest rate paid to investors in the capital. We define R^r as

$$R_{t+1}^r = z_{t+1}\alpha \left(k_{t+1}\right)^{\alpha-1} \quad (15)$$

That is, R^r depends on future productivity, is decreasing in capital, and is uncertain from the investors point of view because of the uncertain *z*. Productivity z is such that the expected return is as modeled in table 1.

Note, that all variables are expressed in real terms - in the units of final output.

4.1 Central Bank and Government

It is assumed that the government pays gross interest on the safe asset, and finances expenditures by taxing the household. The government budget is balanced:

$$g_t = taxes_t = R_{t-1}^s res_{t-1} - res_t \quad (16)$$

The role of the central bank in this economy is limited. Here we allow for a shock to the safe interest rate through the household s Euler equation (11) which is supposed to resemble monetary policy shock.

4.2 Equilibrium

Equilibrium in this model is a set of allocations: $\{c_t, d_t, y_t, k_t, k_t^b, res_t, b_t, \hat{\sigma}_t^2, g_t\}_0^\infty$ such that given prices and beliefs all agents solve their problems and markets clear.

5 Simulations

5.1 Calibration and Parameter Values

In the model, most of the parameters are standard. The only nonstandard parameters are learning costs, a, moment of productivity distribution - $E\left(z\right)$ and initial variance of agents' beliefs, σ_t^2.This group of parameters was selected to ensure the existence of solutions, and non-negative values of information cost, b_t[7]. Also, for alternative learning specifications, to ensure the existence of equilibrium, these three parameters have to be different.

Table 3: Parameter Values

		Linear	Entropy
ρ	risk-aversion	2	
α	capital share	0.33	
δ	depreciation	0.02	
β	discount factor	0.95	
$E\left(z\right)$	mean productivity	10	
a	information costs	1	1.5
σ_t^2	prior variance	1.1	

Table 3 shows the selected parameter values used for the simulation below. In this paper we are focusing mainly on intuition, how low policy rates and / or subdued market volatility can influence risk-taking and what the contribution of the information channel could be. Above, in the section on partial equilibrium, we show that both risk-taking channels work regardless of parameter values. That is why we consider our procedure for selecting information costs and prior variance satisfactory for our purpose. If, however, one is targeting quantitative effects, more rigorous calibration of information costs and market volatility is necessary. For the mean productivity values, we are targeting that the condition $z_{t+1}\alpha\left(k_{t+1}\right)^{\alpha-1} > R_t^s$ is satisfied in the steady state. Even though the selected number seems to be large, it results in steady state risky asset return 1.2369 and 1.0920 for linear and entropy rules respectively.

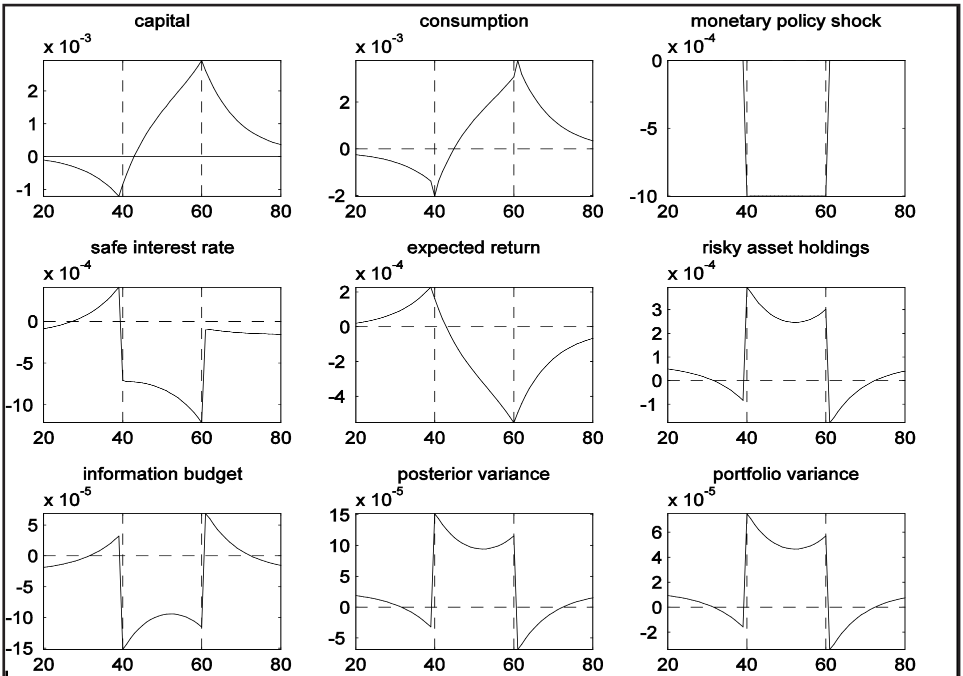
In the next subsection we show general equilibrium results for our model of information acquisition.

7 *Condition for the existence of non-negative bt are in appendix.*

5.2 Simulations

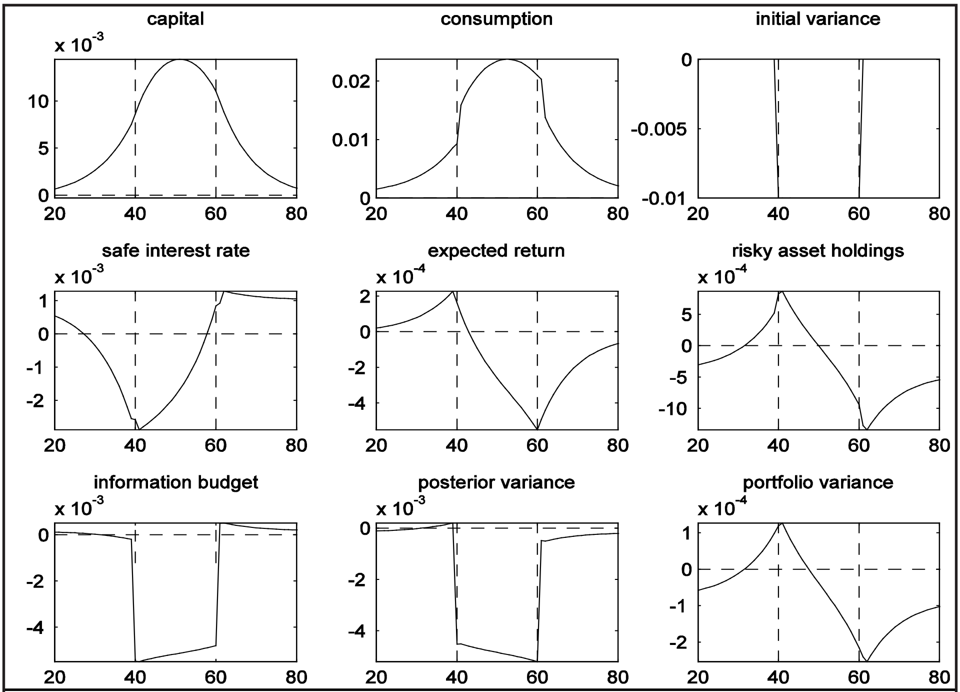
We start with a linear learning rule model. For the simulations[8], we lowered the initial variance or safe interest rate for 1% and 0.1% respectively for 20 periods. The safe interest rate was reduced using a deterministic shock to the household's Euler equation (11). After 20 periods, both of the variables return to their steady state values, together with other model variables. Figure 2 reports responses for a linear learning rule model. The vertical dashed lines mark the start and end of the decline in selected variables. Panel a shows the reaction to a shock to the Euler equation, which we here call "monetary policy". Recall that there is no money in the model, and this name is figurative to suggest that the shock to the safe interest rate resembles monetary authority action in a full-blown New Keynesian model. One also can note from the panel a that agents are rational and the safe interest change is expected: the slight adjustment to the change starts ahead of the actual shock realization. Following the decline in the safe interest rate, the bank's risky asset holdings increase. The risky asset is investment into capital in our economy, which is why additional capital is accumulated. Larger capital accumulation reduces the expected return on capital. This is the force that returns the model to the steady state after the policy is removed. Before this, there is a drop in the information budget as a larger risk premium (expected return on risky asset falls less than safe interest rate) makes an agent tolerate larger risk. Lower information acquisition determines larger posterior variance. Both larger posterior variance and the risky asset position increase the bank's portfolio risk.

Figure 2: Linear Learning Rule



a. Accommodative "Monetary Policy" Shock, 0.1%

8 *The simulations are done using Dynare version 4.2.*

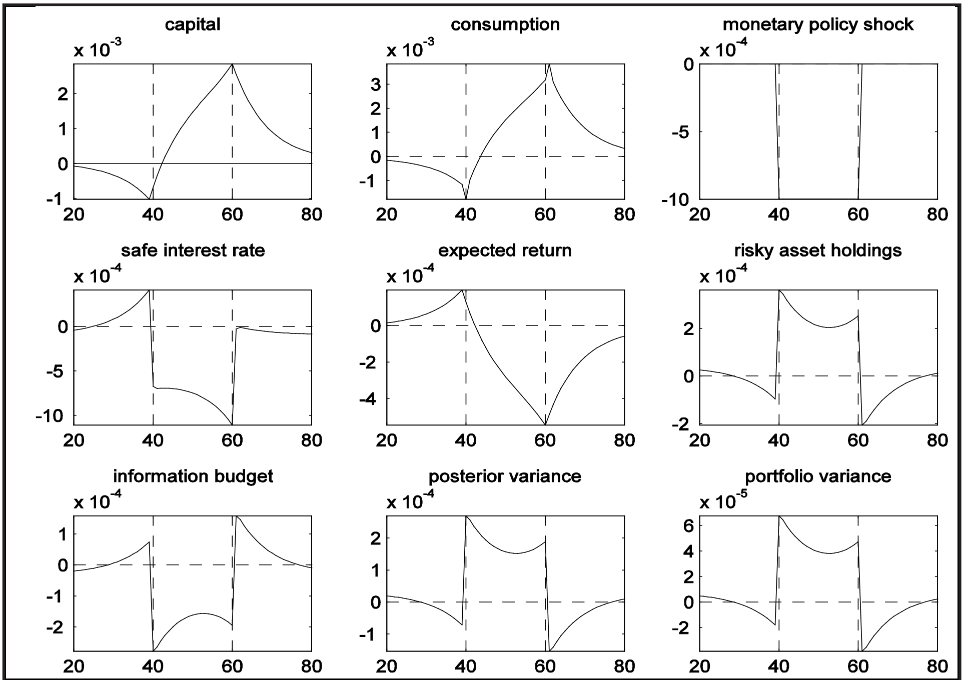


b. Prior Variance Reduction by 1%

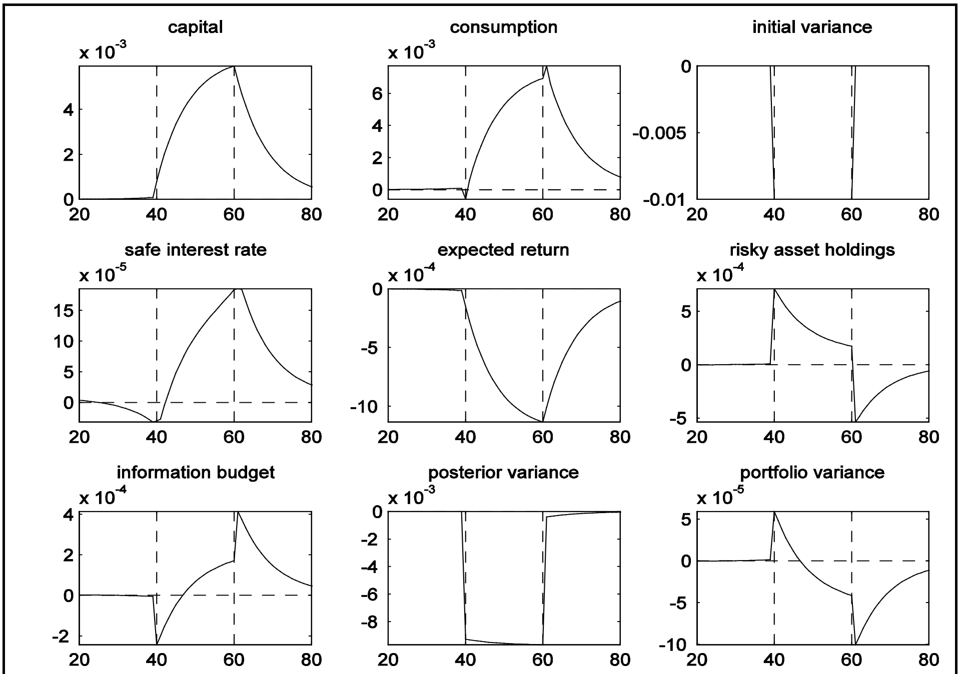
For the change in initial variance, panel b, we also observe some adjustments beforehand. Anticipating decline in the variance, risky-asset holdings, capital and consumption start to increase before the actual variance reduction. Accumulation of capital declines the return on capital, which is the risky asset in our model. At period t =40 when the initial variance falls, the information budget falls too. Posterior variance, being the difference of prior variance and the information budget, declines, but two times less than the prior. Information costs are unity in this model, which is why, without the information channel the posterior variance from (6) should fall by the same amount as the prior variance. A decline in the information budget here reduces the effect of initial volatility on the risk that agents are facing. This and a rise in risky asset portfolio holdings increase portfolio variance above the steady state level. At period t =50, when the expected return reaches its minimum value, risky asset holdings and portfolio variance start declining. After the policy is removed and the level of capital reduced, the increasing expected return returns the economy back to the steady state.

For the model with the entropy learning rule, figure 3, panel a; a very similar response to interest rate decline is found. A reduction in safe interest rates simultaneously reduces information acquisition and increases risky asset holdings. A combination of the two increases the bank's portfolio risk.

Figure 3: Entropy Learning Rule



a. Accommodative "Monetary Policy" Shock, 0.1%



b. Prior Variance Reduction by 1%

When considering a reduction in prior variance, figure 3, panel b, a different response of the information budget and safe interest rate is observed. Risky asset holdings are increased, raising capital and consumption and decreasing the expected return. At the same time there is a reduction in the information budget, but unlike in the linear model, this effect is short-lived, and is reversed in a couple of periods. This leads to short-lived increase in portfolio variance, which declines afterwards. If in the linear model the information budget is always below the steady state level for lower prior variance, it is not the case in entropy. With the entropy constraint, there is a larger effect of falling expected return on the information budget. With the expected return falling, the information budget starts to increase, decreasing posterior variance and portfolio risk. Also, the initial fall in the information budget is less pronounced than in the linear model. The difference is partially attributed to larger information costs and partially to a different functional form of learning function.

Conclusions

This paper addresses the debate as to whether periods of low policy rates and low market volatility could lead to overaccumulation of risky assets. It is motivated by the number of empirical studies showing that increase in risk appetite is associated with low policy rates.

We contribute to the literature by building a model with rationally inattentive financial agents, who decide how much to invest in information acquisition subject to information costs. Information acquisition is modelled as paying for a decline in risky asset variance. We consider two basic learning functions: entropy and linear learning rule.

It is then shown that with a low safe interest rate there are two channels of increase in risk-taking: a standard in the literature search-for-yield, and a decline in the information budget. These two channels result in a high risky asset position and high risk of the asset per se, as an agent face higher uncertainty about asset returns. As a result, agent accumulates more risk in his or her portfolio when the safe asset rate falls. These findings are robust to the learning rule specification.

Another result is larger risk-taking with the decline in risky asset volatility. When the variance of risky return falls, agents rationally increase their risky asset holdings. At the same time, they are willing to pay less for further reduction in return variance. Lower incentives for information acquisition partially offset the drop in initial variance, with posterior variance falling much less than the prior. In combination with larger risky asset holdings, it increases agent's portfolio variance.

Acknowledgements

We are grateful to Sergey Slobodyan, Filip Matejka, Michal Kejak, Mirko Wiederholt, Vincent Sterk and Jaume Ventura for comments and suggestions. The work was supported by Charles University Grant Agency grant number 528314 and published as CERGE-EI Working Paper no 536.

References

ADAM, K. and A. MARCET (2010) *Booms and busts in asset prices*. IMES Discussion Paper Series 10-E-02, Institute for Monetary and Economic Studies, Bank of Japan.

ADRIAN, T., A. ESTRELLA and H. S. SHIN (2010) Monetary cycles, financial cycles, and the business cycle. Technical report.

AHREND, R. (2010) Monetary ease: A factor behind financial crises? Some evidence from OECD countries. *Economics - The Open-Access, Open-Assessment E-Journal* 4(12), 1 30.

BANK FOR INTERNATIONAL SETTLEMENTS (2014) 84th annual report: 1 april 2013 31 march 2014.

BEAUDRY, P., D. NAM, and J. WANG (2011) *Do mood swings drive business cycles and is it rationa*l. Working Paper Series 17651, NBER.

BERNANKE, B. S. (2013, March 1) Long-term interest rates. San Francisco, California. Annual Monetary/Macroeconomics Conference: *The Past and Future of Monetary Policy.*

BORIO, C. and H. ZHU (2008) *Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism?* BIS Working Papers 268, Bank for International Settlements.

BOZ, E. and E. G. MENDOZA (2010) *Financial innovation, the discovery of risk, and the U.S. credit crisis.* NBER Working Papers 16020, National Bureau of Economic Research, Inc.

BULLARD, J., G. EVANS, and S. H. SEPPO (2010) A model of near-rational exuberance. *Macroeconomic Dynamics* 14(2), 166 188.

CABRALES, A., O. GOSSNER and R. SERRANO (2013) Entropy and the value of information for investors. *American Economic Review* 103(1), 360 377.

DELL ARICCIA, G., L. LAEVEN, and R. MARQUEZ (2010) *Monetary policy, leverage, and bank risk-taking.* IMF Working Papers 10/276, International Monetary Fund.

DETKEN, C. and F. SMETS (2004) *Asset price booms and monetary policy*. Working Paper Series 364, European Central Bank.

FUHRER, J. (2011) *The role of expectations in U.S. in ation dynamics.* Working Paper Series 11-11, Federal Reserve Bank of Boston.

GAMBACORTA, L. (2009) Monetary policy and the risk-taking channel. *BIS quarterly review* (December), 43 53.

KURZ, M. and M. MOTOLESE (2010) Diverse beliefs and time variability of risk premia. *Economic Theory* 47(2), 293 335.

LAOPODIS, N. (2010) Dynamic linkages between monetary policy and the stock market. *Review of Quantitative Finance and Accounting* 35(3), 271 293.

LORENZONI, G. (2009) A theory of demand shocks. *American Economic Review* 99(5), 2050 2084.

MACKOWIAK, B. and M. WIEDERHOLT (2009) Optimal Sticky Prices under Rational Inattention. *American Economic Review* 99(3), 769 803.

MADDALONI, A. (2011) Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the U.S. lending standards. *Review of Financial Studies* 24(6), 2121 2165.

MATEJKA, F. and A. MCKAY (2015) Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review* 105(1), 272 298.

NIEUWERBURGH, S. V. and L. VELDKAMP (2010) Information acquisition and underdiversification. *Review of Economic Studies* 77(2), 779 805.

ONGENA, S. and J. PEYDRO (2011) Loose monetary policy and excessive credit and liquidity risk-taking by banks. Technical report.
STEIN, J. C. (2013, February 7) Overheating in credit markets: Origins, measurement, and policy responses. St. Louis, Missouri. A Research Symposium sponsored by the Federal Reserve Bank of St. Louis.
TAYLOR, J. B. (2007) Housing and monetary policy. *Proceedings*, 463 476.
TAYLOR, J. B. (2010) Getting back on track: macroeconomic policy lessons from the financial crisis. *Review* (May), 165 176.
WHITE, W. R. (2012) Ultra easy monetary policy and the law of unintended consequences. Technical report.

Contact address

Mgr. Volha Audzei
CERGE-EI - a joint workplace of Charles University in Prague and the Economics Institute of the Academy of Sciences of the Czech Republic/ CERGE-EI – společné pracoviště Univerzity Karlovy v Praze a Národohospodářského ústavu AV ČR
(e-mail: vaudzei@cerge-ei.cz)

Appendix Comparative static

Linear Learning Rule

From table 2 the solution for information budget is positive when information costs are:

$$a > \frac{(\mu_t - R_t^s)^2}{\sigma_t^2} + 1$$

That is, larger than one plus the expected return to variance ratio. In this interval, the derivative with respect to initial variance is positive:

$$a\left(1 - \frac{1}{2\sqrt{a\left((\mu_t - R_t^s)^2 + \sigma_t^2\right)}}\right) > 0$$

And the derivative with respect to risk premium is non-positive:

$$-\frac{a\left(\mu_t - R_t^s\right)}{\sqrt{a\left((\mu_t - R_t^s)^2 + \sigma_t^2\right)}} < 0$$

The impact of information costs increase is always positive on the interval with positive b_t:

$$\sigma_t^2 - \frac{\sqrt{a\left((\mu_t - R_t^s)^2 + \sigma_t^2\right)}}{2a} > 0$$

The effect on risky asset portfolio holdings is characterized by the following derivatives:

$$\frac{\partial k_t^b}{\partial \sigma_t^2} = -\frac{a^2\left(\mu_t - R_t^s\right)}{2\rho\left(a\left((\mu_t - R_t^s)^2 + \sigma_t^2\right)\right)^{\frac{3}{2}}} < 0$$

$$\frac{\partial k_t^b}{\partial\left(\mu_t - R_t^s\right)} = \frac{a^2\sigma_t^2}{2\rho\left(a\left((\mu_t - R_t^s)^2 + \sigma_t^2\right)\right)^{\frac{3}{2}}} > 0$$

Entropy Learning Rule

b_t, is positive when

$$\log[\frac{a\sigma_t^2}{(\mu_t - R_t^s)^2 + \sigma_t^2}] > \log\left[\log[2]\right] - \log[2] = \log\left[\frac{\log[2]}{2}\right] = -1.0597$$

The derivative of budget with respect to initial variance, σ_t^2 is always nonnegative:

$$\frac{a\left(\mu_t - R_t^s\right)^2}{\sigma_t^2\left(\left(\mu_t - R_t^s\right)^2 + \sigma_t^2\right)\log(4)} \geq 0$$

The derivative with respect to risk premia $\left(\mu_t - R_t^s\right)$ is always non-positive:

$$\frac{a\left(\mu_t - R_t^s\right)}{\left(\left(\mu_t - R_t^s\right)^2 + sigma\right)\log[2]} \leq 0$$

The derivative of budget, b_t, with respect to information costs, a, is:

$$\frac{1 + \log[\frac{a\sigma_t^2}{(\mu_t - R_t^s)^2 + \sigma_t^2}] - \log[\log[4]]}{\log(4)}$$

The sign of the derivative is determined by the nominator. The derivative is positive when:

$$\log[\frac{a\sigma_t^2}{\left(\mu_t - R_t^s\right)^2 + \sigma_t^2}] > \log[\log[4]] - 1 = \log\left[\frac{2\log 2}{e}\right] = -0.6703$$

Since 0.6703 >1.0597, there is a region where the derivative could be negative. The information budget is decreasing with information cost, when information costs are:

$$\frac{\log[2]}{2}\left(\frac{\left(\mu_t - R_t^s\right)^2}{\sigma_t^2} + 1\right) < a < \frac{2\log 2}{e}\left(\frac{\left(\mu_t - R_t^s\right)^2}{\sigma_t^2} + 1\right)$$

Thus for relatively small information costs, an increase in information cost will reduce the information budget. For other, feasible values of a, an increase in information costs also increases the information budget.

The effect on risky asset portfolio holdings is characterized by the following derivatives:

$$\frac{\partial k_t^b}{\partial \sigma_t^2} = -\frac{a\left(\mu_t - R_t^s\right)}{\rho\left(\left(\mu_t - R_t^s\right)^2 + \sigma_t^2\right)^2\log(4)} < 0$$

$$\frac{\partial k_t^b}{\partial(\mu_t - R_t^s)} = \frac{a(-(\mu_t - R_t^s)^2 + \sigma_t^2)}{\rho((\mu_t - R_t^s)^2 + \sigma_t^2)^2\log(4)} > 0$$

if $\sigma_t^2 > \left(\mu_t - R_t^s\right)^2$